Fachbereich Mathematik und Statistik
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Universität Konstanz

Real Algebraic Geometry II

Exercise Sheet 2
Linear orders

Exercise 5
(4 points)
Let $(A, \leq_A)$ be a countable dense linear order without endpoints. Let $(B, \leq_B)$ be an arbitrary countable linear order. Show that $(B, \leq_B)$ is isomorphic to a subordering of $(A, \leq_A)$.
In particular, any countable ordinal embeds into $(\mathbb{Q}, \leq)$.

Exercise 6
(4 points)
Let $(A, \leq)$ be a linear order. Suppose that there exists a countable subset $B \subseteq A$ such that B is dense in A, i.e. for any $a, a' \in A$ with $a < a'$, there exists $b \in B$ with $a \leq b \leq a'$.
Let $C \subseteq A$ be a subset which is well-ordered by $\leq$. Show that C is countable.
In particular, any well-ordered subset of $(\mathbb{R}, \leq)$ *is countable.*

Please hand in your solutions by ***Thursday, 02 May 2019, 10:00h*** *(postbox 14 in F4).*