Fachbereich Mathematik und Statistik
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Universität Konstanz

Real Algebraic Geometry II

Exercise Sheet 9
Fields of generalized power series

Exercise 29
(4 points)
Let k be an Archimedean field and let G be an ordered abelian group. Let $\mathbb{K} = k((G))$.

(a) Find an order-preserving isomorphism of groups from $v(\mathbb{K}^\times)$ to G.

(b) Consider both Archimedean fields k and $\overline{\mathbb{K}}$ as subfields of $\mathbb{R}$. Let

$$s = \sum_{g \in G} s(g) t^g \in R_v \setminus \overline{0}.$$

Show that for the residue $\overline{s}$ of s we have $\overline{s} = s(v_{\min}(s))$.

(c) Conclude that $\overline{\mathbb{K}} = k$.

Exercise 30
(4 points)
Let k be an Archimedean field which is square root closed for positive elements, i.e. for any $a \in k^{>0}$, there exists $b \in k$ with $b^2 = a$. Let G be an ordered abelian group which is 2-divisible, i.e. for any $g \in G$, there exists $h \in G$ such that $h + h = g$. Let $\mathbb{K} = k((G))$.

(a) Let $\varepsilon \in \mathbb{K}$ with support$(\varepsilon) \subseteq G^{>0}$.

(i) Let $\alpha \in \mathbb{Q}^{>0}$. Show that

$$\sum_{n=0}^{\infty} \frac{(\alpha)_n}{n!} \varepsilon^n \in \mathbb{K},$$

where

$$(\alpha)_n = \prod_{k=0}^{n-1} (\alpha - k).$$

(ii) Show that

$$\left(\sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}\right)_n}{n!} \varepsilon^n \right)^2 = 1 + \varepsilon.$$

(b) Deduce that $\mathbb{K}$ is square root closed for positive elements

*Please hand in your solutions by **Friday, 21 June 2019, 10:00h** (postbox 14 in F4).*