Fachbereich Mathematik und Statistik
Prof. Dr. Salma Kuhlmann
Lothar Sebastian Krapp
Simon Müller
SoSe 2019

Universität Konstanz

Real Algebraic Geometry II

Exercise Sheet 12
Valuations on power series fields

Exercise 37
(16 points)
Let $G \neq \{0\}$ be a divisible ordered abelian group and let $H_2 \subseteq G$ be a non-trivial proper convex subgroup.

(a) Show that for some divisible ordered abelian group H_1, there exists an order-preserving isomorphism

$$\varphi\colon G \to H,$$

where $H = H_1 \times H_2$ is ordered lexicographically. Moreover, show that $\varphi(H_2)$ is a convex subgroup of H.

(Hint: Consider G as a $\mathbb{Q}$-vector space.)

(b) Let $K = \mathbb{R}((H_2))$. Find an order-preserving isomorphism of fields

$$\psi\colon \mathbb{R}((H)) \to K((H_1)).$$

(c) Let w be the valuation on $\mathbb{R}((H))$ associated to the convex subgroup $\varphi(H_2)$ of H.

(i) Find an order-preserving embedding of groups from the value group $w(\mathbb{R}((H))^\times)$ into $(\mathbb{R}((H))^{>0}, \cdot, 1, <)$.

(ii) Find an order-preserving embedding of fields from the the residue field $\mathbb{R}((H))w$ into $\mathbb{R}((H))$.

(Hint: You may find it useful to identify $\mathbb{R}((H))$ with $K((H_1))$ via the isomorphism ψ. Moreover, recall Exercise 35.)

(d) Describe explicitly the residue map

$$\mathbb{R}((H))_w \to \mathbb{R}((H))w, s \mapsto sw.$$

Please hand in your solutions by ***Thursday, 11 July 2019, 10:00h*** *(postbox 14 in F4).*