Universität Konstanz

Logic Colloquium

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Tarski's Exponential Function Problem

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A mathematical structure is called *decidable* if there exists an algorithm which for a given statement in a formal logical language decides whether this statement is true or false in the structure. Tarski gave in [2] an explicit decision algorithm for the real numbers in the language $(+, \cdot, 0, 1, <)$. He then asked whether one can still find such a decision algorithm if a symbol exp for exponentiation is added to the language. This question is known as *Tarski's Exponential Function Problem* and remains unsolved to the date. However, progress has been made in [1], where the authors prove that the answer to Tarski's question is positive if one assumes Schanuel's Conjecture – a famous open conjecture from Transcendental Number Theory.

Tarski's decision algorithm is originally based on geometrical arguments. But, as it only uses certain properties – say axioms – of the real numbers, it also gives rise to another algorithm which uses purely recursion theoretic arguments. In the first part of my talk, I will briefly describe Tarski's decision algorithm and contrast the two different kinds of decision algorithms. In the second part, I will outline the conditional proof of the existence of a decision algorithm for the language $(+, \cdot, 0, 1, <, \exp)$, putting particular emphasis on the logical arguments.

All relevant notions and terminology will be introduced during the talk.

References:

[1] A. Macintyre and A. Wilkie, 'On the decidability of the real exponential field', *Kreiseliana: about and around Georg Kreisel* (ed. P. Odifreddi; A. K. Peters, Wellesley, MA, 1996) 441–467.

[2] A. Tarski, *A decision method for elementary algebra and geometry* (RAND Corporation, Santa Monica, CA, 1948).