O-minimal Exponential Fields and Real Exponentiation

Lothar Sebastian Krapp
Universität Konstanz

Abstract

An ordered exponential field is an ordered field \((K, +, \cdot, 0, 1, <)\) equipped with a unary function \(\exp\) which is an order-preserving isomorphism from \((K, +, 0, <)\) to \((K^{\cdot 0}, \cdot, 1, <)\) (see [2]). The most prominent example of an ordered exponential field is \(\mathbb{R}_{\exp}\), the ordered field of real numbers with its standard exponential function. Tarski asked the question whether \(\mathbb{R}_{\exp}\) is decidable (see [4]); this question remains unsolved to the date. However, it has been shown by Macintyre and Wilkie in [3] that the answer to Tarski’s question is positive if one assumes Schanuel’s Conjecture — an open conjecture from transcendental number theory. Their work was based on [5], in which Wilkie proves that \(\mathbb{R}_{\exp}\) is o-minimal. In [1], Berarducci and Servi draw further connections between the decidability question of the real exponential field and general o-minimal exponential fields, and hence motivate the study of the class of o-minimal exponential fields.

In my talk I will firstly give an introduction to o-minimal exponential fields and present some of their algebraic, model theoretic and valuation theoretic properties (see [2]). Secondly I will explain how these properties are related to the decidability problem of \(\mathbb{R}_{\exp}\) and Schanuel’s Conjecture.

All model theoretic and valuation theoretic notions will briefly be introduced during the talk.

References


