O-minimal Exponential Fields and Real Exponentiation

Lothar Sebastian Krapp

Universität Konstanz

Abstract

An ordered exponential field is an ordered field $(K, +, \cdot, 0, 1, <)$ equipped with a unary function exp which is an order-preserving isomorphism from $(K, +, 0, <)$ to $(K^{>0}, \cdot, 1, <)$ (see [2]). The most prominent example of an ordered exponential field is $\mathbb{R}_{\exp}$, the ordered field of real numbers with its standard exponential function. Tarski asked the question whether $\mathbb{R}_{\exp}$ is decidable (see [4]); this question remains unsolved to the date. However, it has been shown by Macintyre and Wilkie in [3] that the answer to Tarski's question is positive if one assumes Schanuel's Conjecture – an open conjecture from transcendental number theory. Their work was based on [5], in which Wilkie proves that $\mathbb{R}_{\exp}$ is o-minimal. In [1], Berarducci and Servi draw further connections between the decidability question of the real exponential field and general o-minimal exponential fields, and hence motivate the study of the class of o-minimal exponential fields.

In my talk I will firstly give an introduction to o-minimal exponential fields and present some of their algebraic, model theoretic and valuation theoretic properties (see [2]). Secondly I will explain how these properties are related to the decidability problem of $\mathbb{R}_{\exp}$ and Schanuel's Conjecture.

All model theoretic and valuation theoretic notions will briefly be introduced during the talk.

References

[1] A. Berarducci and T. Servi, 'An effective version of Wilkie's theorem of the complement and some effective o-minimality results', *Ann. Pure Appl. Logic* 125 (2004) 43–74.

[2] S. Kuhlmann, *Ordered Exponential Fields*, Fields Inst. Monogr. 12 (Amer. Math. Soc., Providence, RI, 2000).

[3] A. Macintyre and A. Wilkie, 'On the decidability of the real exponential field', *Kreiseliana: about and around Georg Kreisel* (ed. P. Odifreddi; A. K. Peters, Wellesley, MA, 1996) 441–467.

[4] A. Tarski, *A decision method for elementary algebra and geometry* (RAND Corporation, Santa Monica, CA, 1948).

[5] A. Wilkie, 'Model completeness results for expansions of the ordered Field of real numbers by restricted Pfaffian functions and the exponential function', *J. Amer. Math. Soc.* 9 (1996) 1051–1094.