Real Exponentiation and Exponential Groups

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Abstract

An exponential exp on an ordered field $(K, +, \cdot, 0, 1, <)$ is an order-preserving isomorphism from $(K, +, 0, <)$ to $(K^{>0}, \cdot, 1, <)$. The structure $(K, +, \cdot, 0, 1, <, \exp)$ is called an ordered exponential field. A group exponential h on an ordered group $(G, +, 0, <)$ is an order-preserving bijection from the value set $v_G(G)$ of G under the natural valuation to $G^{<0}$, and the pair (G, h) is called an exponential group. Any exponential on an ordered field K which is compatible with the natural valuation v induces a group exponential on the value group $v(K)$ of K. A complete characterisation of countable exponential groups which are induced by countable ordered exponential fields is given in [2].

The most prominent example of an ordered exponential field is the real exponential field $(\mathbb{R}, +, \cdot, 0, 1, <, \exp)$, where exp is the standard exponential function $x \mapsto \mathrm{e}^x$. Models of the theory of the real exponential field $T_{\exp}$ exhibit nice model theoretic and geometric properties which are due to o-minimality (cf. [3]). These can be exploited for the study of exponential groups induced by models of $T_{\exp}$.

The aim of my talk will be to present a full characterisation of countable exponential groups which are induced by models of $T_{\exp}$ (cf. [1]). All model theoretic and valuation theoretic notions will briefly be introduced during the talk.

References

[1] L. S. Krapp, 'Value Groups and Residue Fields of Models of Real Exponentiation', Preprint, 2018, arXiv:1803.03153.

[2] S. Kuhlmann, *Ordered Exponential Fields*, Fields Inst. Monogr. 12 (Amer. Math. Soc., Providence, RI, 2000).

[3] A. Wilkie, 'Model completeness results for expansions of the ordered Field of real numbers by restricted Pfaffian functions and the exponential function', *J. Amer. Math. Soc.* 9 (1996) 1051–1094.