# Schanuel's Conjecture and Roots of Exponential Polynomials

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**Abstract**

Schanuel's Conjecture (SC) was first mentioned in the literature by Stephen Schanuel's doctoral supervisor Serge Lang in [3]. It states as follows:

*Let* $\alpha_1, \ldots, \alpha_n \in \mathbb{C}$ *be linearly independent over* $\mathbb{Q}$. *Then the transcendence degree of* $\mathbb{Q}(\alpha_1, \ldots, \alpha_n, \exp(\alpha_1), \ldots, \exp(\alpha_n))$ *over* $\mathbb{Q}$ *is at least* $n$.

(SC) has numerous algebraic and number theoretic implications; for instance, the open question whether $\pi$ and e are algebraically independent over $\mathbb{Q}$ would be answered positively under the assumption of this conjecture. But also in other mathematical areas, such as model theory, there is a wide range of results which are based on the assumption of (SC).

In my talk, I will sketch how (SC) implies the decidability of $T_{\exp}$, the theory of the real exponential field $(\mathbb{R}, +, \cdot, 0, 1, <, \exp)$ (cf. [4]). To this end, the most crucial argument leading to the application of (SC) is the fact that the decidability of $T_{\exp}$ only depends on the existence of an effective procedure which decides whether a given exponential polynomial has a root in $\mathbb{R}$.

If time permits, I will also outline some connections to general o-minimal ordered exponential fields (cf. [1, 2]).

All model theoretic notions will briefly be introduced during the talk.

## References

[1] S. Kuhlmann, *Ordered Exponential Fields*, Fields Inst. Monogr. 12 (Amer. Math. Soc., Providence, RI, 2000).

[2] L. S. Krapp, 'O-minimal exponential fields and their residue fields' (extended abstract), *Oberwolfach Reports* 13 (2016) 3357–3359, doi:10.4171/OWR/2016/60.

[3] S. Lang, *Introduction to transcendental numbers*, Addison-Wesley, Reading, Mass., 1966).

[4] A. Macintyre and A. Wilkie, 'On the decidability of the real exponential field', *Kreiseliana: about and around Georg Kreisel* (ed. P. Odifreddi; A. K. Peters, Wellesley, MA, 1996) 441–467.