Real Exponentiation and Exponential Integer Parts

Lothar Sebastian Krapp

Universität Konstanz

Abstract

Let K be an ordered field. An integer part of K is a discretely ordered subring $Z \subseteq K$ such that for any $a \in K$ there exists a unique $b \in Z$ with $b \leq a < b+1$. Integer parts carry certain information about the algebraic structure of the corresponding field; for instance, both the value group and the residue field under any convex valuation on an ordered field are determined by an integer part. In turn, one can also ask what conditions on a discretely ordered ring Z ensure that Z can be realised as the integer part of certain ordered fields. Most prominently, Shepherdson [2] gave a complete characterisation of discretely ordered rings which are the integer parts of real closed fields.

In my joint work with Carl [1], we investigate what model theoretic conditions on a discretely ordered ring Z ensure that Z is an integer part of a model of real exponentiation, i.e. of an ordered field $(K, +, \cdot, 0, 1, <)$ with a unary function E such that $(K, +, \cdot, 0, 1, <, E)$ is elementarily equivalent to $(\mathbb{R}, +, \cdot, 0, 1, <, 2^x)$.

In my talk, I will explain how a model of Peano Arithmetic M can be used to construct a real closed field K_M supporting an exponential such that $Z_M = M \cup (-M)$ is an integer part of K_M. Moreover, I will outline the main steps of the proof of our main result: If $(M, +, \cdot, 0, 1, <)$ is elementarily equivalent to $(\mathbb{N}, +, \cdot, 0, 1, <)$, then Z_M is an integer part of a model of real exponentiation.

References

[1] M. Carl and L. S. Krapp, 'Models of true arithmetic are integer parts of models of real exponentiation', *J. Log. Anal.*, to appear.

[2] J. C. Shepherdson, 'A Non-standard Model for a Free Variable Fragment of Number Theory', *Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys.* 12 (1964) 79–86.