Definable henselian valuations by conditions on the value group

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Abstract

Let K be a field. A valuation ring $\mathcal{O}$ on K is a subring of K such that for any $x \in K^\times$ we have $x \in \mathcal{O}$ or $x^{-1} \in \mathcal{O}$. To each valuation ring, there is an associated ordered abelian group G and a surjective map $v\colon K^\times \to G$. The map v is called a valuation on K with value group G and valuation ring $\mathcal{O}$. Valuations satisfying a certain condition on roots of polynomials are called henselian. Henselianity entails further tame algebraic and model theoretic properties.

From a model theoretic point of view, one is particularly interested in the definability of henselian valuation rings in the first-order language of rings $\mathcal{L}_\mathrm{r} = \{+, -, \cdot, 0, 1\}$. This study has been motivated by decidability questions and, more recently, by the classification of NIP fields (see [1]). In our work, we also consider henselian valuation rings on ordered fields that are definable in the language of ordered rings $\mathcal{L}_\mathrm{or} = \mathcal{L}_\mathrm{r} \cup \{<\}$ (see [2]).

In my talk, I will firstly outline the basic concept of henselian valuations and motivate the study of definable henselian valuation rings in fields and ordered fields. Secondly, I will present topological conditions on the value group that ensure the definability of the corresponding valuation ring independent of the specific field (see [3]).

All valuation and model theoretic notions will briefly be introduced during the talk.

References

[1] A. Fehm and F. Jahnke, 'Recent progress on definability of Henselian valuations', *Ordered Algebraic Structures and Related Topics*, Contemp. Math. 697 (eds F. Broglia, F. Delon, M. Dickmann, D. Gondard-Cozette and V. A. Powers; Amer. Math. Soc., Providence, RI, 2017), 135–143.

[2] L. S. Krapp, S. Kuhlmann and G. Lehéricy, 'Ordered fields dense in their real closure and definable convex valuations', *Forum Math.* 33 (2021) 953–972.

[3] L. S. Krapp, S. Kuhlmann and M. Link, 'Definability of henselian valuations by conditions on the value group', Preprint, 2021, arXiv:2105.09234v1.