SEMINAR IN STABILITY THEORY

Stability theory is part of model theory, it is very beautiful and elegant. It gives a very abstract way of analyzing a large class of structures. In some sense it is a generalization of algebraic geometry: it enables us to talk about independence, dimension, generic elements, etc, in a more general setting.
The material goes beyond what is typically offered in an introductory class in model theory.
We shall start with Morley's famous categoricity theorem that states that if T is a complete first order theory that has a unique model up to isomorphism in some uncountable cardinal, then it has a unique model up to isomorphism in every uncountable cardinal. These theories are called uncountably categorical theories.
Uncountable categorical theories are the simplest examples of stable theories, and this will lead us to the next topics.
We will develop the basic notions of stability theory: the order property, indiscernibles, definability of types, forking, imaginary elements and more. We may also look at stable groups.
We shall see examples of stable theories from algebra such as algebraically closed fields.

Tentative week by week outline

(1) Reminder of Model theory and beginning of Morely's Theorem
  (a) Languages, Structures, Elementary extensions, Compactness, Lowenheim-Skolem theorem.
  (b) Indiscernible sequences – 15 of [1].
(2) ω-stable theories – 16 of [1].
(3) Prime extensions – 17 of [1].
(4) Lachlan's theorem – 18 of [1].
(5) Vaughtian Pairs – 19 of [1].
(6) Algebraic formulas – 20 of [1].
(7) Strongly minimal sets – 21 of [1].
(8) The Baldwin-Lachlan proof of Morley's theorem – 22 of [1].
(9) The Monster model, heirs and co-heirs – 23, 32 of [1].
(10) Stability – 33 of [1].
(11) Definabe types – 34 of [1].
(12) Elimination of imaginaries and T^{eq} – 35 of [1].

References

[1] Katrin Tent and Martin Ziegler. *A Course in Model Theory*. work in progress.