Universität Konstanz

Fachbereich
Mathematik und Statistik

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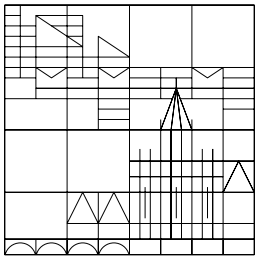

Real Algebraic Geometry II
Exercise Sheet 5

Exercise 1
Let $\Gamma \neq \emptyset$ be a totally ordered set.
Show that

(a) there is a well ordered cofinal subset $A \subseteq \Gamma$.

(b) Γ has no last element if and only if A has no last element.

(c) if Γ has no last element then the order type of A is a limit ordinal.

Hint: (a) can be shown by transfinite induction.

Exercise 2
Consider valued vector spaces (V_1, v_1) and (V_2, v_2). Let $h : (V_1, v_1) \xrightarrow{\sim} (V_2, v_2)$ be a value preserving isomorphism. Let $S = \{a_\rho\}_{\rho \in \lambda}$ be a pseudo-Cauchy sequence in (V_1, v_1).

(a) Show that $S' := \{h(a_\rho)\}_{\rho \in \lambda}$ is a pseudo-Cauchy sequence in (V_2, v_2).

(b) Let x be a pseudo-limit of S in (V_1, v_1). Show that $h(x)$ is a pseudo-limit of S' in (V_2, v_2).

(c) Conclude that (V_1, v_1) is pseudo-complete if and only if (V_2, v_2) is pseudo-complete.

Exercise 3
Consider a Q-valued vector space (V, v). Let $S = \{a_\rho\}_{\rho\in\lambda}$ be a pseudo-Cauchy sequence in (V, v) with pseudo-limit $\alpha \in V$. Let $q \in Q \setminus \{0\}$ and let $x \in V$.

(a) Show that $\{q \cdot a_\rho\}_{\rho\in\lambda}$ is a pseudo-Cauchy sequence with pseudo limit $q \cdot \alpha$.

(b) Show that $\{x+a_\rho\}_{\rho\in\lambda}$ is a pseudo-Cauchy sequence with pseudo limit $x+\alpha$.

(c) Show that if 0 is a pseudo-limit of $\{x+q \cdot a_\rho\}_{\rho\in\lambda}$, then $-\frac{x}{q}$ is a pseudo-limit of S.

The exercise will be collected **Thursday, 21/05/2015** until 10.00 at box 13 near F 441.
