Fields of Generalized Power Series
(the mysteries and obstacles around formal summability)

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Abstract: Fields of generalized (formal) power series (with exponents in totally ordered *abelian* groups) are obtained via formal constructions, widely generalizing those of the classical Laurent or Puiseux series. They are natural (pseudo) completions of fields of fractions of group rings. They play a central role in many areas of mathematics, logic, and theoretical computer science, e.g. in commutative algebra, (real) algebraic geometry, asymptotic analysis, asymptotic differential algebra, dynamical systems, model theory and automata theory, to mention just a few. A crucial issue (and often a major tool or obstacle) concerns the formal summability of families of formal series. We will focus on this aspect and illustrate by examples how it is handled while formally defining algebraic, analytic, difference and differential operators on these objects. Although the objects under consideration are commutative, we hope that our contribution could provide thoughts for non-commutative versions.