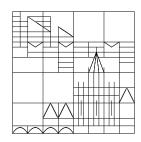
Universität Konstanz Fachbereich Mathematik und Statistik Daniel Plaumann Summer 2015



CLASSICAL ALGEBRAIC GEOMETRY

1st problem sheet Tutorial on 23 April 2015

- Show that any two ordered sets of *n* + 2 points in general position in Pⁿ are projectively equivalent. ([Ha], Ex. 1.6)
- 2. Let Γ be a finite subset of \mathbb{P}^n . Show that if Γ consists of d points and is not contained in a line, then Γ may be described by polynomials of degree at most d 1.

([*Ha*], *Ex.* 1.3)

- **3.** Show that if seven points $p_1, \ldots, p_7 \in \mathbb{P}^3$ lie on a twisted cubic *C*, then *C* is the zero locus of all quadratic forms vanishing at p_1, \ldots, p_7 . ([*Ha*], *Ex.* 1.13)
- 4. Let *k* be any integer between 1 and d 1. Show that the set of points $[Z_0, \ldots, Z_d]$ in \mathbb{P}^d for which the $(d k + 1) \times (k + 1)$ -matrix

$\begin{bmatrix} Z_0 \end{bmatrix}$	Z_1	Z_2	•	Z_{k-1} Z_k
Z_1	Z_2	•		$\cdot Z_{k+1}$
Z_2		•	•	
	•	•	•	
	•	•	•	$\cdot Z_{d-1}$
Z_{d-k}	•	•		Z_{d-1} Z_d

has rank 1 is precisely the rational normal curve.

- 5. Show that the rational normal curve passing through d + 3 points in \mathbb{P}^d constructed in the lecture is unique.
- 6. Let $\Gamma = \{p_0, \dots, p_{d+2}\}, \Gamma' = \{p'_0, \dots, p'_{d+2}\}$ be two ordered sets of points in \mathbb{P}^d . Let $v_d: \mathbb{P}^1 \to \mathbb{P}^d$ (resp. v'_d) be the unique rational normal curve passing through Γ (resp. Γ') and put $\Delta = v_d^{-1}(\Gamma), \Delta' = (v'_d)^{-1}(\Gamma')$.

Show that Γ and Γ' are projectively equivalent in \mathbb{P}^d if and only if Δ and Δ' are projectively equivalent in \mathbb{P}^1 .

How is the latter characterized in terms of cross-ratios? ([Ha], Ex. 1.19)