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Summer 2015

CLASSICAL ALGEBRAIC GEOMETRY

1st problem sheet
Tutorial on 23 April 2015

1. Show that any two ordered sets of $n+2$ points in general position in $\mathbb{P}^n$ are projectively equivalent. *([Ha], Ex. 1.6)*

2. Let Γ be a finite subset of $\mathbb{P}^n$. Show that if Γ consists of d points and is not contained in a line, then Γ may be described by polynomials of degree at most $d-1$. *([Ha], Ex. 1.3)*

3. Show that if seven points $p_1, \dots, p_7 \in \mathbb{P}^3$ lie on a twisted cubic C, then C is the zero locus of all quadratic forms vanishing at $p_1, \dots, p_7$. *([Ha], Ex. 1.13)*

4. Let k be any integer between 1 and $d-1$. Show that the set of points $[Z_0, \dots, Z_d]$ in $\mathbb{P}^d$ for which the $(d-k+1) \times (k+1)$-matrix

$$\begin{bmatrix} Z_0 & Z_1 & Z_2 & \cdot & Z_{k-1} & Z_k \\ Z_1 & Z_2 & \cdot & \cdot & \cdot & Z_{k+1} \\ Z_2 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & Z_{d-1} \\ Z_{d-k} & \cdot & \cdot & \cdot & Z_{d-1} & Z_d \end{bmatrix}$$

has rank 1 is precisely the rational normal curve.

5. Show that the rational normal curve passing through $d+3$ points in $\mathbb{P}^d$ constructed in the lecture is unique.

6. Let $\Gamma = \{p_0, \dots, p_{d+2}\}$, $\Gamma' = \{p'_0, \dots, p'_{d+2}\}$ be two ordered sets of points in $\mathbb{P}^d$. Let $\nu_d \colon \mathbb{P}^1 \to \mathbb{P}^d$ (resp. ν'_d) be the unique rational normal curve passing through Γ (resp. Γ') and put $\Delta = \nu_d^{-1}(\Gamma)$, $\Delta' = (\nu'_d)^{-1}(\Gamma')$.

   Show that Γ and Γ' are projectively equivalent in $\mathbb{P}^d$ if and only if Δ and Δ' are projectively equivalent in $\mathbb{P}^1$.

   How is the latter characterized in terms of cross-ratios? *([Ha], Ex. 1.19)*