Universität Konstanz
Fachbereich Mathematik und Statistik
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CLASSICAL ALGEBRAIC GEOMETRY

2nd problem sheet
Tutorial on 30 April 2015

1. Take the lecture notes from your first course in Algebraic Geometry or your favourite text book and convince yourself that the notion of *morphism* between quasi-projective varieties agrees with the one given in the lecture.

2. Let $X \subset \mathbb{P}^m$ be a projective variety and $\varphi\colon X \to \mathbb{P}^n$ a morphism. Show that the graph

$$\Gamma_\varphi = \{(x, \varphi(x)) : x \in X\} \subset X \times \mathbb{P}^n$$

is closed.

Does the converse hold, i.e. is every map with closed graph a morphism?

(*[Ha], Ex. 2.24*)

3. Prove the second version of the fundamental theorem of elimination theory (Thm. 2.3 on the lecture slides) by adapting the proof of the first version (Thm. 2.1). If you get stuck, look in Harris for hints.

4. Let C be the twisted cubic in $\mathbb{P}^3$. Compute the projection of C from the point $[1, 0, 0, 1]$ and from $[0, 1, 0, 0]$ onto suitable hyperplanes. (Note that taking resultants may not be the most efficient way of doing this.) (*[Ha], Ex. 3.8*)