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Fachbereich Mathematik und Statistik
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CLASSICAL ALGEBRAIC GEOMETRY

5th problem sheet
Tutorial on 19 May 2015

1. Show that the image of the rational normal curve $C \subset \mathbb{P}^n$ under the projection π_p from the point $p = [1, 0, \ldots, 0] \in C$ is a rational normal curve in $\mathbb{P}^{n-1}$.
For the twisted cubic, compare this with Exercise 4 on sheet #2. *([Ha], Ex. 7.7)*

2. Consider the rational map

$$\varphi\colon \begin{cases} \mathbb{P}^2 & \dashrightarrow & \mathbb{P}^2 \\ [X_0, X_1, X_2] & \mapsto & [X_1X_2, X_0X_2, X_0X_1] \end{cases},$$

called the *(standard) quadratic Cremona transformation*.

Show that φ is birational and equal to its inverse.

Determine the domain of φ and an open subset U of $\mathbb{P}^2$ with $\varphi(U) \subset U$.

Describe the geometry of the quadratic Cremona transformation: Which lines does it contract? What does this have to do with the points in which it is undefined?

3. Let C be the cubic curve in $\mathbb{P}^2$ defined by the equation

$$ZY^2 = X^3 + X^2Z.$$

Show that the projection of C from the point $p = [0, 0, 1]$ is a birational isomorphism of C with $\mathbb{P}^1$, so that C is a rational curve. *([Ha], Ex. 7.12)*