CLASSICAL ALGEBRAIC GEOMETRY

6th problem sheet
Tutorial on 26 May 2015

1. Compute the proper transform of the plane curve $\mathcal{V}(y^2 - x^3) \subset \mathbb{A}^2$ in the blow-up of $\mathbb{A}^2$ at the origin.

2. Assume $\operatorname{char}(K) \neq 2$ and let $X \subset \mathbb{P}^n$ be a quadric hypersurface defined by a quadratic form $Q \in K[Z_0, \ldots, Z_n]_2$.
   Show that the singular points of X form a linear subspace. What is its dimension?

3. Let $X \subset \mathbb{C}^n$ be a smooth algebraic curve over $\mathbb{C}$. Let $\Delta \subset \mathbb{C}$ be an open neighbourhood of 0 and
   $$\varphi \colon \Delta \to \mathbb{C}^n$$
   a holomorphic map with $\varphi(\Delta) \subset X$ and $\varphi'(0) \neq 0$.

   (1) Show that the tangent space to X at $\varphi(0)$ is spanned by the vector $\varphi'(0)$.
   (2) Use this to compute the tangent space to the twisted cubic in $\mathbb{P}^3$ at any point. Verify directly that the definition from the lecture gives the same result.