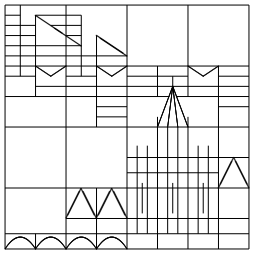
Universität Konstanz
Fachbereich Mathematik und Statistik
Daniel Plaumann
Summer 2015



CLASSICAL ALGEBRAIC GEOMETRY

7th problem sheet
Tutorial on 2 June 2015

1. Assume $\mathrm{char}(K) = 2$. Show that all projective tangent lines to the plane curve defined by $XY - Z^2$ pass through a common point in $\mathbb{P}^2$.

2. Let X be a quasi-projective variety. Show that the function

$$\begin{cases} X & \to & \mathbb{N} \\ p & \mapsto & \dim T_p(X) \end{cases}$$

is upper-semicontinuous on X. This means that

$$\left\{ p \in X : \dim T_p(X) \leqslant k \right\}$$

is Zariski-open in X, for any $k \geqslant 0$.

3. Use Bertini's theorem to give another proof of the fact that the general hypersurface of degree d in $\mathbb{P}^n$ is smooth.