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Summer 2015

CLASSICAL ALGEBRAIC GEOMETRY

9th problem sheet
Tutorial on 16 June 2015

1. Let $I, J \subset K[Z_0, \dots, Z_n]$ be two homogeneous ideals.
Show that the following are equivalent:

(1) I and J have the same saturation.
(2) There exists $d \geqslant 0$ such that $I_m = J_m$ holds for all $m \geqslant d$.
(3) I and J generate the same ideal *locally*, i.e. they are the same in every localization $K[Z_0/Z_i, \dots, Z_n/Z_i]$, $i = 0, \dots, n$. *([Ha], Ex. 5.2)*

2. Find the Hilbert polynomial of a hypersurface of degree d in $\mathbb{P}^n$ and verify in this way that the dimension of a hypersurface is indeed $n - 1$. *([Ha], Ex. 13.5)*

3. Find the Hilbert polynomial of the Segre variety $\Sigma_{m,n} = \sigma(\mathbb{P}^m \times \mathbb{P}^n) \subset \mathbb{P}^{(m+1)(n+1)-1}$ and verify in this way that the dimension of this variety is indeed $m + n$.
([Ha], Ex. 13.6)