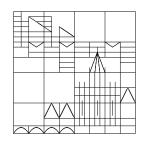
Universität Konstanz Fachbereich Mathematik und Statistik Daniel Plaumann Summer 2015



CLASSICAL ALGEBRAIC GEOMETRY

13th problem sheet Tutorial on 14 July 2015

- 1. Let L_1, L_2 be two disjoint lines in \mathbb{P}^3 and let $p \in \mathbb{P}^3 \setminus (L_1 \cup L_2)$. Show that there exists a unique line through p that intersects both L_1 and L_2 .
- **2.** Assume char(K) = 0. Let $\Gamma = \{p_1, \ldots, p_6\} \subset \mathbb{P}^2$, where

$$p_1 = [1, 0, 0], p_2 = [0, 1, 0], p_3 = [0, 0, 1], p_4 = [1, 3, 1], p_5 = [2, -1, 1], p_6 = [-1, -2, 1].$$

Use software for the following:

- (a) Show that the points in Γ are in general position, i.e. no three are on a line and not all six on a conic.
- (b) Show that the space $I(\Gamma)_3$ of cubics passing through Γ has dimension 4.
- (c) Find four cubics F_0 , F_1 , F_2 , F_3 spanning $I(\Gamma)_3$ and show that the image of the rational map

$$\varphi: \left\{ \begin{array}{ccc} \mathbb{P}^2 & \cdots & \mathbb{P}^3 \\ [Z] & \mapsto & [F_0(Z), F_1(Z), F_2(Z), F_3(Z)] \end{array} \right.$$

is a smooth cubic surface in \mathbb{P}^3 .

3. Let $U \subset \mathbb{P}K[Z_0, ..., Z_4]_5$ be the open set of smooth quintic threefolds (hypersurfaces of degree 5 in \mathbb{P}^4). Consider the incidence correspondence

$$\Sigma = \{ (L, X) \in \mathbb{G}(1, 4) \times U \colon L \text{ is a line in } X \}.$$

You may assume that Σ is irreducible.

- (a) Show that Σ has the same dimension as U (which is 125).
- (b) Show that the hypersurface $\mathcal{V}(Z_0^5 + \dots + Z_4^5) \subset \mathbb{P}^4$ contains infinitely many lines.
- (c) What, if anything, does (a) show about lines on a quintic threefold?