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CLASSICAL ALGEBRAIC GEOMETRY

13th problem sheet
Tutorial on 14 July 2015

1. Let L_1, L_2 be two disjoint lines in $\mathbb{P}^3$ and let $p \in \mathbb{P}^3 \setminus (L_1 \cup L_2)$. Show that there exists a unique line through p that intersects both L_1 and L_2.

2. Assume $\operatorname{char}(K) = 0$. Let $\Gamma = \{p_1, \dots, p_6\} \subset \mathbb{P}^2$, where

$$p_1 = [1,0,0], p_2 = [0,1,0], p_3 = [0,0,1], p_4 = [1,3,1], p_5 = [2,-1,1], p_6 = [-1,-2,1].$$

Use software for the following:

(a) Show that the points in Γ are in general position, i.e. no three are on a line and not all six on a conic.

(b) Show that the space $I(\Gamma)_3$ of cubics passing through Γ has dimension 4.

(c) Find four cubics F_0, F_1, F_2, F_3 spanning $I(\Gamma)_3$ and show that the image of the rational map

$$\varphi\colon \begin{cases} \mathbb{P}^2 & \dashrightarrow & \mathbb{P}^3 \\ [Z] & \mapsto & [F_0(Z), F_1(Z), F_2(Z), F_3(Z)] \end{cases}$$

is a smooth cubic surface in $\mathbb{P}^3$.

3. Let $U \subset \mathbb{P}K[Z_0, \dots, Z_4]_5$ be the open set of smooth quintic threefolds (hypersurfaces of degree 5 in $\mathbb{P}^4$). Consider the incidence correspondence

$$\Sigma = \left\{(L, X) \in \mathbb{G}(1,4) \times U \colon L \text{ is a line in } X\right\}.$$

You may assume that Σ is irreducible.

(a) Show that Σ has the same dimension as U (which is 125).

(b) Show that the hypersurface $\mathcal{V}(Z_0^5 + \cdots + Z_4^5) \subset \mathbb{P}^4$ contains infinitely many lines.

(c) What, if anything, does (a) show about lines on a quintic threefold?