# Some applications of the tangency variety

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Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ be a polynomial function. The set

$$T_f := \{x \in \mathbb{R}^n | \exists \lambda \in \mathbb{R} : \nabla f(x) = \lambda x\}$$

is called the *tangency variety* of $f$.
Let us denote by $B_\infty(f)$ the set of *critical values of singularities at infinity* and $K_\infty(f)$ the set of *asymptotic critical values* of $f$.
Set

$$R_\infty(f, T_f) := \{t \in \mathbb{R} | \exists x_k \to \infty, x_k \in T_f, f(x_k) \to t\}.$$

Then

$$B_\infty(f) \subset \mathbb{R}_\infty(f, T_f) \subset K_\infty(f)$$

and $\inf_{x \in \mathbb{R}^n} f(x) = \inf_{x \in T_f} f(x)$.

In this talk I shall give some applications of the tangency variety in the following problems:

- Computing the Łojasiewicz exponent at infinity;
- minimizing polynomial functions via SDP;
- characterizing critical values of singularities at infinity of a polynomial function on a smooth algebraic surface in $\mathbb{R}^n$.