Contents

Introduction **1**

1 Global solutions to wave equations — existence theorems **7**

2 L^p–L^q-decay estimates for the linear wave equation **15**

3 Linear symmetric hyperbolic systems **21**
3.1 Energy estimates 22
3.2 A global existence theorem 28
3.3 Remarks on other methods 31

4 Some inequalities **34**

5 Local existence for quasilinear symmetric hyperbolic systems **57**

6 High energy estimates **78**

7 Weighted a priori estimates for small data **82**

8 Global solutions to wave equations — proofs **89**
8.1 Proof of Theorem 1.1 89
8.2 Proof of Theorem 1.2 93

9 Other methods **104**

10 Development of singularities **108**

11 More evolution equations **112**
11.1 Equations of elasticity 114
11.1.1 Initially isotropic media in $\mathbb{R}^3$ 114
11.1.2 Initially cubic media in $\mathbb{R}^2$ 121
11.2 Heat equations 137
11.3 Equations of thermoelasticity 144
11.4 Schrödinger equations 177
11.5 Klein–Gordon equations 182
11.6 Maxwell equations 191
11.7 Plate equations 197

12 Further aspects and questions **207**

13 Evolution equations in waveguides **221**
13.1 Nonlinear wave equations 222
13.1.1 Linear part 222
13.1.2 Nonlinear part 238
13.2 Schrödinger equations 240
13.3 Equations of elasticity and Maxwell equations 243
13.4 General waveguides 254

Appendix **271**
A Interpolation 271
B The Theorem of Cauchy–Kowalevsky 278
C A local existence theorem for hyperbolic-parabolic systems 282

References **289**

Notation **302**

Index **304**

Introduction

Many problems arising in the applied sciences lead to nonlinear initial value problems (nonlinear Cauchy problems) of the following type

$$V_t + AV = F(V, \ldots, \nabla^\beta V), \quad V(t=0) = V^0.$$

Here $V = V(t,x)$ is a vector-valued function taking values in $\mathbb{R}^k$ (or $\mathbb{C}^k$), where $t \geq 0$, $x \in \mathbb{R}^n$, and A is a given linear differential operator of order m with k, n, $m \in \mathbb{N}$. F is a given nonlinear function of V and its derivatives up to order $|\beta| \leq m$, and ∇ denotes the gradient with respect to x, while V^0 is a given initial value. In particular the case $|\beta| = m$, i.e. the case of fully nonlinear initial value problems, is of interest.
An important example from mathematical physics is the wave equation describing an infinite vibrating string (membrane, sound wave, respectively) in $\mathbb{R}^1$ ($\mathbb{R}^2$, $\mathbb{R}^3$, respectively; generalized: $\mathbb{R}^n$). The second-order differential equation for the elongation $y = y(t,x)$ at time t and position x is the following:

$$y_{tt} - \nabla' \frac{\nabla y}{\sqrt{1 + |\nabla y|^2}} = 0,$$

where ∇' denotes the divergence. This can also be written as

$$y_{tt} - \Delta y = \nabla' \frac{\nabla y}{\sqrt{1 + |\nabla y|^2}} - \Delta y =: f(\nabla y, \nabla^2 y).$$

We notice that f has the following property:

$$f(W) = \mathcal{O}(|W|^3) \quad \text{as } |W| \to 0.$$

Additionally one has prescribed initial values

$$y(t=0) = y_0, \quad y_t(t=0) = y_1.$$

The transformation defined by $V := (y_t, \nabla y)$ turns the nonlinear wave equation for y into a first-order system for V as described above. The investigation of such nonlinear evolution equations has found an increasing interest in the last years, in particular because of their application to the typical partial differential equations arising in mathematical physics.
We are interested in the existence and uniqueness of global solutions, i.e. solutions $V = V(t,x)$ which are defined for all values of the time parameter t. The solutions will be smooth solutions, e.g. C^1-functions with respect to t taking values in Sobolev spaces of sufficiently high order of differentiability. In particular they will be classical solutions. Moreover we wish to describe the asymptotic behavior of the solutions as $t \to \infty$.

It is well known for the nonlinear wave equation, the first example above, that in general one cannot expect to obtain a global smooth solution. That is to say, the solution may develop singularities in finite time, no matter how smooth or how small the initial data are. This phenomenon is known for more general nonlinear hyperbolic systems and also for many other systems from mathematical physics, biology, etc., including the systems which are mentioned below. Therefore, a general global existence theorem can only be proved under special assumptions on the nonlinearity and on the initial data. The result will be a theorem which is applicable for small initial data, assuming a certain degree of vanishing of the nonlinearity near zero. The necessary degree depends on the space dimension, being a weaker assumption for higher dimensions. This is strongly connected with the asymptotic behavior of solutions to the associated linearized system ($F \equiv 0$ resp. $f \equiv 0$ in the example above) as $t \to \infty$, which gives a first insight into the means used for the proof.
Further examples of nonlinear evolution equations which can be written in the general first-order form after a suitable transformation are the following. They will be discussed in more detail in Chapter 11.

- Equations of elasticity:

$$\partial_t^2 U_i = \sum_{m,j,k=1}^{n} C_{imjk}(\nabla U)\partial_m \partial_k U_j, \quad i = 1, \ldots, n,$$

$$U(t=0) = U^0, \quad U_t(t=0) = U^1.$$

  We shall discuss the homogeneous, initially isotropic case for $n = 3$ and the homogeneous, initially cubic case for $n = 2$.

- Heat equations:

$$u_t - \Delta u = F(u, \nabla u, \nabla^2 u), \quad u(t=0) = u_0.$$

- Equations of thermoelasticity:

$$\partial_t^2 U_i = \sum_{m,j,k=1}^{n} C_{imjk}(\nabla U, \theta)\partial_m \partial_k U_j + \tilde{C}_{im}(\nabla U, \theta)\partial_m \theta, \quad i = 1, 2, 3,$$

$$(\theta + T_0)a(\nabla U, \theta)\partial_t \theta = \nabla' q(\nabla U, \theta, \nabla\theta) + tr\{\tilde{C}_{km}(\nabla U, \theta)'_{km} \cdot (\partial_t \partial_s U_r)_{rs}\}(\theta + T_0),$$

$$U(t=0) = U^0, \quad U_t(t=0) = U^1, \quad \theta(t=0) = \theta^0.$$

  The homogeneous, initially isotropic case will be discussed here.

- Schrödinger equations:

$$u_t - \mathrm{i}\Delta u = F(u, \nabla u), \quad u(t=0) = u_0.$$

- Klein–Gordon equations:

$$y_{tt} - \Delta y + my = f(y, y_t, \nabla y, \nabla y_t, \nabla^2 y), \quad m > 0,$$

$$y(t=0) = y_0, \quad y_t(t=0) = y_1.$$

- Maxwell equations:

$$\begin{aligned} D_t - \nabla \times H &= 0, \\ B_t + \nabla \times E &= 0, \end{aligned}$$

$$\begin{array}{cc} D(t=0) = D^0, & B(t=0) = B^0, \\ \nabla' D = 0, & \nabla' B = 0, \\ D = \varepsilon(E), & B = \mu(H). \end{array}$$

- Plate equations:

$$y_{tt} + \Delta^2 y = f(y_t, \nabla^2 y) + \sum_{i=1}^{n} b_i(y_t, \nabla^2 y)\partial_i y_t,$$

$$y(t=0) = y_0, \quad y_t(t=0) = y_1.$$

In order to obtain existence theorems to these systems, we shall apply the classical method of continuing local solutions (local with respect to t), provided a priori estimates are known. The proof of the a priori estimates represents the non-classical part of the approach. It requires ideas and techniques which mainly have been developed in the last years, in particular the idea of using the decay of solutions to the associated linearized problems. These new techniques were essential to overcome the difficulties in the study of *fully* nonlinear systems, i.e. systems where the nonlinearity involves the highest derivatives appearing on the linear left-hand side. We remark that in this sense the Schrödinger equations and the plate equations above are not fully nonlinear. The highest derivatives that appear in the nonlinearity can still directly be dominated by the linear part in the energy estimates, see Chapter 11.
The general method by which all the systems mentioned before can be dealt with (cum grano salis) is described by the following scheme.
We discuss the system

$$V_t + AV = F(V, \ldots, \nabla^\beta V), \quad V(t=0) = V^0,$$

where F is assumed to be smooth and to satisfy

$$F(W) = \mathcal{O}(|W|^{\alpha+1}) \quad \text{as } |W| \to 0, \qquad \text{for some } \alpha \in \mathbb{N}.$$

The larger α is, the smaller is the impact that the nonlinearity will have for small values of $|W|$, i.e. the linear behavior will dominate for some time and there is some hope that it will lead to global solutions for sufficiently small data if the linear decay is strong enough. This will depend on the space dimension.
The general scheme consists of the following Steps **A**–**E**.

A: Decay of solutions to the linearized system:

A solution V to the associated linearized problem

$$V_t + AV = 0, \quad V(t=0) = V^0,$$

satisfies

$$\|V(t)\|_q \le c(1+t)^{-d}\|V^0\|_{N,p},$$

where $2 \le q \le \infty$ (or $2 \le q < \infty$), $1/p + 1/q = 1$; $c, d > 0$ and $N \in \mathbb{N}$ are functions of q and of the space dimension n. (E.g. for the wave equation above: $d = \frac{n-1}{2}(1 - \frac{2}{q})$.) This is usually proved by using explicit representation formulae and/or the representation via the Fourier transform.

B: Local existence and uniqueness:

There is a local solution V to the nonlinear system on some time interval $[0,T]$, $T > 0$, with the following regularity:

$$V \in C^0([0,T], W^{s,2}) \cap C^1([0,T], W^{\tilde{s},2}),$$

where $s, \tilde{s} \in \mathbb{N}$ are sufficiently large to guarantee a classical solution. The proof of a local existence theorem is always a problem itself. We shall present the proof of the corresponding theorem for the wave equation in detail.

C: High energy estimates:

The local solution V satisfies

$$\|V(t)\|_{s,2} \le C\|V^0\|_{s,2} \cdot \exp\left\{ C\int_0^t \|V(r)\|_{b,\infty}^{\alpha}\, dr \right\}, \quad t \in [0,T].$$

C only depends on s, not on T or V^0. b is independent of s, that is, the exponential term does not involve higher derivatives in the L^∞-norm (which allows to close the circle in Step **E**). This inequality is proved using general inequalities for composite functions (see Chapter 4).

D: Weighted a priori estimates:

The local solution satisfies

$$\sup_{0 \le t \le T} (1+t)^{d_1}\|V(t)\|_{s_1,q_1} \le M_0 < \infty,$$

where M_0 is independent of T, s_1 is sufficiently large, $q_1 = q_1(\alpha)$ is chosen appropriately for each problem and $d_1 = d(q_1, n)$ according to **A**, provided V^0 is sufficiently small (in a sense to be made precise later; roughly, high Sobolev norms of V^0 are small).

In this step the information obtained in **A** is exploited with the help of the classical formula

$$V(t) = \mathrm{e}^{-tA}V^0 + \int_0^t \mathrm{e}^{-(t-r)A}F(V, \dots, \nabla^\beta V)(r)dr,$$

where $\mathrm{e}^{-tA}V^0$ symbolically stands for the solution to the linearized problem with initial value V^0.

E: Final energy estimate:

The results in **C** and **D** easily lead to the following a priori bound:

$$\|V(t)\|_{s,2} \leq K\|V^0\|_{s,2}, \quad 0 \leq t \leq T,$$

$s \in \mathbb{N}$ being sufficiently large, V^0 being sufficiently small and K being independent of T. This a priori estimate allows us to apply now the standard continuation argument and to continue the local solution obtained in Step **B** to a solution defined for all $t \in [0, \infty)$.

The method described above immediately provides information on the asymptotic behavior of the global solution as $t \to \infty$ in Step **D** and in Step **E**.

This general scheme applies to all the above systems mutatis mutandis; for example, there may appear certain derivatives with respect to t of V in the integrand of the exponential in Step **C**. Moreover the nonlinearity may depend on t and x explicitly. Nevertheless, difficult questions can arise in the discussion of the details for each specific system. Particularly interesting are the necessary modifications that have to be made for the equations of thermoelasticity. This system cannot directly be put into the framework just described because it consists of different types of differential equations (hyperbolic, parabolic), and also different types of nonlinearities appear which exclude for example a uniform sharp estimate as in Step **A**. Instead different components of V have to be dealt with in different ways. Altogether however, global existence theorems will again be proved in the spirit of the Steps **A**–**E**.

This underlines the generality of the approach. Of course, this generality prevents the results from being optimal in some cases. We shall discuss this in detail for the following general wave equation:

$$y_{tt} - \Delta y = f(y_t, \nabla y, \nabla y_t, \nabla^2 y),$$

$$y(t=0) = y_0, \quad y_t(t=0) = y_1.$$

For this we shall go through the Steps **A–E** in Chapters 1–8. Moreover, a more or less optimal result is presented, the proof of which uses invariance properties of the d'Alembert operator $\partial_t^2 - \Delta$ under the generators of the Lorentz group. The other examples will be studied in Chapter 11. In several of the cases there, these subtle invariances are not available.

To underline the necessity of studying conditions under which small data problems allow global solutions we shall shortly describe some blow-up results — results on the development of singularities in finite time even for small data — in Chapter 10. In Chapter 9 a few other methods are briefly mentioned and Chapter 12 tries to outline some recent developments and future projects going beyond the main line of this book.

The scheme described above can be found in [94]. Similar ideas are present in [117, 119, 158, 178].

One may think of the global existence results as a kind of stability result for small perturbations of the associated linear problems. Of course it is of great interest to study solutions for large data but this is beyond the scope of this book. We refer the interested reader to the literature [138, 179, 180, 186]. We also remark that there are much more results on semilinear systems. The emphasis in this book lies on fully nonlinear systems.

In the second edition, we shall treat in the new Chapter 13 linear and nonlinear initial-boundary value problems in waveguides, giving insight into the impact of the geometry of domains with boundaries, and, simultaneously, demonstrating that following the steps **A–E** also here applies, mutatis mutandis.

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