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Introduction

The equations of thermoelasticity describe the elastic and the thermal behavior of elastic, heat conductive media, in particular the reciprocal actions between elastic stresses and temperature differences. We consider the classical thermoelastic system where the elastic part is the usual second-order one in the space variable. The equations are a coupling of the equations of elasticity and of the heat equation and thus build a hyperbolic-parabolic system. Indeed, both hyperbolic and parabolic effects are encountered. This book discusses the mathematical questions arising in the study of initial value problems and of initial boundary value problems to these equations, both for linear and for nonlinear systems. Classical boundary conditions of the Dirichlet type — rigidly clamped, constant temperature — or the Neumann type — traction free, insulated — are considered, as well as the linearized equations together with contact boundary conditions.

It is known both for hyperbolic and for parabolic nonlinear equations and systems that global smooth solutions in general might not exist but that a blow-up may occur. The criteria according to which global solutions still exist are different for hyperbolic and for parabolic equations. Hence the question arises whether the behavior will be dominated by the hyperbolic or by the parabolic part. The answer will depend on the number of space dimensions. This also applies to the question of asymptotic behavior of solutions to the linearized system, where the behavior will also depend on the space dimension, or to be more precise, one dimension on one side and two or three dimensions on the other side.

The methods used for obtaining global existence theorems for *small* data consist of proving suitable a priori estimates, where one often exploits the decay of solutions to the linearized equations. This requires a precise analysis of the asymptotic behavior of such solutions as time tends to infinity, which will finally allow us to describe the asymptotic behavior of solutions to the nonlinear systems as well.

We are mainly interested in proving the well-posedness in the class of smooth solutions and in describing the asymptotic behavior of the solutions as time tends to infinity. This will be possible in the linear case and also in the nonlinear case provided the nonlinearities and the data satisfy certain conditions. Otherwise, a blow-up in finite time has to be expected as examples will show; then weak solutions must be considered.

In one space dimension the picture is almost complete. Bounded or unbounded intervals representing the reference configuration can be dealt with for all the classical boundary conditions. The asymptotic behavior is known, the decay rates are known to be optimal (in the case of absence of forces and heat supplies). For small data global smooth solutions to the nonlinear system will exist; large data lead to a blow-up.

In two or three space dimensions generic nonlinear cases are understood, although there are unsolved problems. Local well-posedness is known in most cases, but concerning global solutions or blow-up for nonlinear situations, only the Cauchy problem and the bounded radially symmetric case have been investigated. This corresponds to the fact that the dynamics in the linear case is complicated apart from these situations, as we shall describe in detail.

Although the system to be considered is a rather special one, it should be stressed that the methods employed are rather general and have been or can be used for purely hyperbolic or parabolic problems.

The aim of the book is to present a state of the art in the treatment of initial value problems and of initial boundary value problems both in linear and nonlinear thermoelasticity. Although well-posedness in the linear theory has been studied for years, the description of the general dynamical system with its asymptotic behavior as time tends to infinity and, in particular, the study of nonlinear systems only started in the late sixties and the early eighties, respectively, and led to very interesting features.

After a brief derivation of the equations in Chapter 1 and the discussion of the well-posedness of the linearized problems in Chapter 2, the asymptotic behavior of solutions to the linearized models in one space dimension is described for bounded and unbounded domains in Chapter 3, up to a result on the propagation of singularities. Chapter 4 examines the corresponding two- and three-dimensional systems for radially symmetric bounded domains and for some Cauchy problems with certain symmetries, while the generic result for bounded domains will be slow decay. The local existence of solutions to the original nonlinear problem is obtained rather generally in Chapter 5. Turning to the question of well-posedness, Chapter 6 discusses the picture in one space dimension in detail for bounded and unbounded domains, for stationary forces, giving global small solutions, while large data lead to a blow-up; in the latter case weak solutions are obtained in special situations. Chapter 7 considers the existence of global solutions in the spatially two- or three-dimensional situation both for the Cauchy problem and for the initial boundary value problem in those domains with symmetry that allowed a detailed description of the time-asymptotic behavior in the linearized case; a blow-up result for the Cauchy problem is also presented. Chapter 8 presents solutions to contact problems, where the linearized equations in several space dimensions together with the typical contact boundary conditions are studied, leading to some global existence theorems and a description of the asymptotic behavior both for quasi-static and fully dynamical situations. Chapter 9 briefly presents some related results, such as the behavior in the case of additional damping effects, the asymptotic behavior in space and a survey of some numerical results.

The topics chosen in this book and the references provided are of course not exhaustive. Nevertheless, we hope that the material presented will help serve as a general reference in the field.

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This supplemental bibliography is a collection of selected articles on other thermoelastic systems such as thermoelastic plates or thermoviscoelastic systems and on control theory for thermoelastic systems. The interested reader might use these references as starting points into related subjects.