HEAT CONDUCTION IN ELASTIC SYSTEMS: FOURIER VERSUS CATTANEO

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ABSTRACT

The classical model for heat conduction using Fourier's law for the relation between the heat flux and the gradient of the temperature qualitatively yields exponentially stable systems for bounded reference configurations. This kind of stability remains the same if one replaces Fourier's law by Cattaneo's (Maxwell's,...) law. Considering thermal and, simultaneously, elastic effects, this similarity with respect to exponential stability remains the same for classical second-order thermoelastic systems, one being a hyperbolic-parabolic coupling, the other being a fully hyperbolic system. The similarities even extend to the asymptotical behavior of solutions to corresponding nonlinear systems. But for thermoelastic plates, a system of fourth order and being one of recently found examples, the picture changes drastically, i.e., this thermoelastic system changes its behavior from an exponentially stable to a non-exponentially stable one, while changing Fourier's law to Cattaneo's law. This raises the question of the "right" modeling.

We present a large class of general systems with this behavior demonstrating that it might be more likely that Fourier and Cattaneo predict different qualitative behavior in thermoelastic systems. For this purpose we consider a coupled system depending on a family of parameters, a special case of which describes the thermoelastic plates above, where in only one singular case of the parameters the exponential stability property is kept while replacing Fourier by Cattaneo - and lost in all other cases. Interestingly, the singular case corresponds to the second-order thermoelastic system mentioned above.

We use methods from functional analysis for proving the loss of exponential stability in the Cattaneo case.

INTRODUCTION

The simplest classical equations of heat conduction for the temperature difference $\theta = \theta(t,x) = T(t,x) - T_0$, where T denotes the absolute temperature and T_0 is a fixed constant reference temperature, and for the heat flux $q = q(t,x)$, and putting constants equal to one w.l.o.g, are given by

$$\begin{aligned} \theta_t(t,x) + \operatorname{div} q(t,x) &= 0, \qquad t \geq 0,\, x \in \mathbb{R}^n, &(1)\\ q(t,x) + \nabla\theta(t,x) &= 0, \qquad \text{(Fourier's law)}. &(2) \end{aligned}$$

Combining (1) and (2) yields the parabolic heat equation equation

$$\theta_t(t,x) - \Delta\theta(t,x) = 0. \tag{3}$$

Observation: Model (3) predicts an infinite speed of propagation of signals.

If one replaces Fourier's law (2) by Cattaneo's (Maxwell's, Vernotte's,...) law,

$$\tau\, q_t(t,x) + q(t,x) = -\nabla\theta(t,x), \tag{4}$$

with a small relaxation parameter $\tau > 0$, and if we combine now (1) with (4) we obtain the hyperbolic damped wave equation

$$\tau\theta_{tt}(t,x) + \theta_t(t,x) - \Delta\theta(t,x) = 0. \tag{5}$$

Observation: Model (5) predicts a finite speed of propagation of signals.

As it is well known solutions to the classical heat equation equation (3) and to the classical damped wave equation (5) qualitatively show the same behavior: In bounded reference configurations Ω, with, for example, zero boundary conditions for θ modeling constant temperature T_0 at the boundary, we have exponential stability of the system as time tends to infinity:

$$\int_\Omega \theta^2(t,x)\,dx \leq C\,\mathrm{e}^{-dt}$$

for some positive constants C, d. E.g. for solutions to (3), we have

$$\int_\Omega \theta^2(t,x)\,dx \leq \tilde{C}\,\mathrm{e}^{-dt}\int_\Omega \theta^2(0,x)\,dx$$

with a constant $\tilde{C}$ being independent of the data $\theta(0,\cdot)$ at time zero ($\tilde{C}$ depends on the domain Ω, essentially on the smallest eigenvalue of the negative Dirichlet-Laplace operator $-\Delta_D$ realized in L^2).

We remark that if we replace the bounded reference configuration Ω by all of $\mathbb{R}^n$ or by an exterior domain, we have similar polynomial (only) decay instead of exponential decay for the L^2-norm or, for example, uniformly in x :

$$\sup_{x\in\mathbb{R}^n} |\theta(t,x)| \leq C\,t^{-n/2}.$$

Now taking into account both elastic and thermal effects, in a bounded reference configuration in one space dimension like $\Omega = (0,1)$, we consider the classical system of thermoelasticity, where $u = u(t,x) = X(t,x) - x$ denotes the displacement (with position $X(t,x)$ at time t for the particle x in the fixed reference configuration Ω), and where again most constants are set equal to one,

$$\begin{aligned} u_{tt} - u_{xx} + \theta_x &= 0, \\ \theta_t + q_x + u_{tx} &= 0, \\ \tau q_t + q + \theta_x &= 0. \end{aligned} \tag{6}$$

Typical boundary conditions are $u(t,0) = u(t,1) = 0$ and $\theta(t,0) = \theta(t,1) = 0$ on the boundary of Ω, modeling the situation where the boundary is kept fixed and where the temperature is again kept constantly equal to the reference temperature T_0.

The qualitative behavior is the same for $\tau = 0$ (Fourier) and for $\tau > 0$ (Cattaneo): As time tends to infinity, the energy tends to zero exponentially. Here the energy at time t means the expression

$$\int_0^1 (u_t^2 + u_x^2 + \theta^2 + q^2)(t,x)\,dx$$

involving kinetic and potential energy in terms of u as well as thermal energy expressed in terms of θ and q.

Even the *quantitative* behavior is similar: Replacing (6) by the corresponding equations with real physically given material constants for different real materials, the optimal $\alpha > 0$ in

$$\int_0^1 (u_t^2 + u_x^2 + \theta^2 + q^2)(t,x)\,dx \le C\,\mathrm{e}^{-\alpha t}$$

is similar for materials like silicon, aluminum alloy, steel, germanium, gallium arsenide, indium arsenide, copper and diamond [1]. Similarities also extend to corresponding nonlinear systems.

These observations of similar behavior raises the impression that both heat conduction models, the Fourier model or the Cattaneo model, lead to the same qualitative (or even quantitative) behavior.

We will present examples with essentially different qualitative behavior, where the system is exponentially stable for the Fourier model, and not exponentially stable for the Cattaneo model. This raises the question of the "right" modeling. We start with models for thermoelastic plates, a system of fourth order in the space variable, and mention a Timoshenko type system where the Cattaneo model even "destroys" an exponential stability given in the model without heat conduction, or with heat conduction modeled by Fourier's law. As main part and new contribution, we present a large class of more general coupled systems with this different behavior. For this purpose we consider a coupled system depending on a family of parameters, a special case of which describes the thermoelastic plates above, where in only one singular case of the parameters the exponential stability property is preserved while replacing Fourier by Cattaneo - and lost in all other cases. This demonstrates that it might be more often that Fourier and Cattaneo predict different qualitative behavior in thermoelastic systems.

The singular case where exponential stability is true for both models corresponds to the second-order thermoelastic system (6).

THERMOELASTIC PLATES

A Kirchhoff type thermoelastic plate can be modeled in a bounded reference configuration $\Omega \subset \mathbb{R}^n$ by the following three equations

$$\begin{aligned} u_{tt} + a\,\Delta^2 u + b\,\Delta\theta &= 0, \\ \theta_t + c\,\mathrm{div}\,q - d\,\Delta u_t &= 0, \\ \tau q_t + kq + \nabla\theta &= 0, \end{aligned} \tag{7}$$

where $u = u(t,x)$ denotes the displacement, $\theta = \theta(t,x)$ the temperature difference, and $q = q(t,x)$ the heat flux again, and a, b, c, d, k are positive constants. For suitable boundary conditions, i.e. the hinged boundary conditions $u(t,\cdot) = \Delta u(t,\cdot) = 0$ on the boundary of Ω, the system is exponentially stable for $\tau = 0$ [3, 4, 5, 6], but not for $\tau > 0$ [7, 8]. That is, the asymptotic behavior for the different models is, surprisingly, essentially different.

We remark that on a formal level *exponential stability of a system* means that, after rewriting the differential equations as a first-order system in time for some vector function $V = V(t,x)$,

$$V_t(t,\cdot) + \mathbb{A}V(t,\cdot) = 0,$$

the semigroup $\{\mathrm{e}^{-\mathbb{A}t}\}_{t\ge 0}$ generated by the differential operator $\mathbb{A}$ acting in the x-variable and being defined in some Hilbert space H is an exponentially stable semigroup. The latter now means that there is a number $\alpha > 0$ and a constant $C_0 > 0$ such that for all initial data V_0 in H, and for all $t \ge 0$ one has

$$\|\mathrm{e}^{-\mathbb{A}t}V_0\|_H \le C_0\,\mathrm{e}^{-\alpha t}\|V_0\|_H.$$

TIMOSHENKO BEAMS

In models for beams of Timoshenko type, a given exponentially stability triggered by a typical memory (history) term, is preserved by adding heat conduction in form of the Fourier model, but is lost – hence "destroyed" – by the Cattaneo model. The four differential equations in the model are given by

$$\begin{aligned} \rho_1\varphi_{tt} - k(\varphi_x + \psi_x)_x &= 0, \\ \rho_2\psi_{tt} - b\psi_{xx} + \int_0^\infty \mathrm{e}^{-s}\psi_{xx}(t-s,\cdot)ds + & \\ k(\varphi_x + \psi) + \delta\theta_x &= 0, \\ \rho_3\theta_t + q_x + \delta\psi_{tx} &= 0, \\ \tau q_t + dq + \theta_x &= 0. \end{aligned}$$

Here, the functions ϕ and ψ model the transverse displacement of a beam with reference configuration $(0,1)$ respectively the rotation angle of a filament. θ and q denote again the temperature difference and the heat flux, respectively. The material constants $\rho_1, \rho_2, k, b, \delta, \rho_3, d$ are positive, as well as the already introduced relaxation parameter τ. The term $\int_0^\infty e^{-s}\psi_{xx}(t-s,\cdot)ds$ models the additional consideration of the history.

Assuming the (academic, in general physically not satisfied) condition

$$\frac{\rho_1}{k} = \frac{\rho_2}{b},$$

which corresponds to the equality of the wave speeds for ϕ and ψ, we have the following picture:

For $\delta = 0$, it is a hyperbolic system with history term for (ϕ, ψ), and exponential stability is given. For the coupled system with $\delta \neq 0$ and $\tau = 0$ (Fourier), the exponential stability is preserved. But for $\delta \neq 0$, $\tau > 0$ (Cattaneo), the exponential stability is lost [2]. Again the question of an appropriate modeling comes up.

α-β-SYSTEMS

We now present a new, larger class of coupled systems where the same effect shows up – exponential stability under the Fourier law, and no exponential stability under the Cattaneo law. It will appear as an abstract α-β-system (10), with parameters $0 \leq \alpha,\ \beta \leq 1$, for functions $u, \theta : [0,\infty) \to \mathcal{H}$ into a Hilbert space $\mathcal{H}$.

The case $\tau = 0$ has been studied before:

$$\begin{aligned} u_{tt}(t) + aAu(t) - bA^\beta\theta(t) &= 0, \\ \theta_t(t) + cA^\alpha\theta(t) + dA^\beta u_t(t) &= 0. \end{aligned} \qquad (8)$$

Here A denotes a self-adjoint operator with countable system of eigenfunctions $(\phi_j)_j$ with corresponding increasing eigenvalues $0 < \lambda_j \to \infty$ as $j \to \infty$. The constants a, b, c, d are positive.

The example of the thermoelastic plate (7) is, for $\tau = 0$, a special case with

$$\alpha = \beta = \frac{1}{2}, \quad A = (-\Delta_D)^2,$$

where $-\Delta_D$ denotes the Dirichlet-Laplace operator with zero (Dirichlet) boundary conditions realized in L^2. The case $\alpha = 1$, $\beta = \frac{1}{2}$ corresponds to the second-order thermoelastic system (6), and with the case $\alpha = 0$, $\beta = \frac{1}{2}$ one can model a viscoelastic system [9].

This system was introduced in [9, 10]. The most detailed recent discussion concerning exponential stability, smoothing properties and more can be found in [11, 12]. Exponential stability is known for (8) in the striped region

$$\mathcal{A}_{es}(\tau = 0) := \{(\beta, \alpha) \mid 1 - 2\beta \leq \alpha \leq 2\beta, \alpha \geq 2\beta - 1\}, \quad (9)$$

see Figure 1.

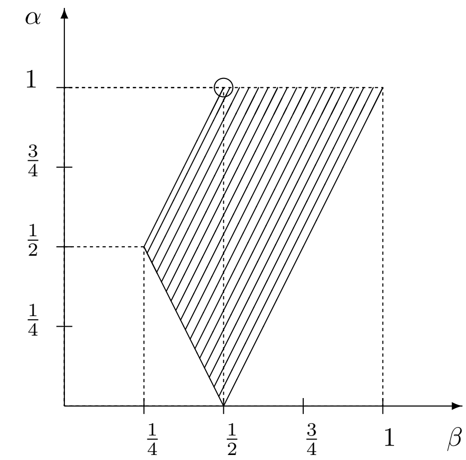

Figure 1: $\mathcal{A}_{es}(\tau = 0)$

The pair $(\beta, \alpha) = (\frac{1}{2}, 1)$ is highlighted by a circle since this will be the only pair for which the exponential stability will remain true if we replace the Fourier law by the Cattaneo law in (10). In the remaining part of the striped region of exponential stability for $\tau = 0$, the property of exponential stability will be lost, see Theorem 1 below.

The abstract Cattaneo version corresponding to the Fourier version (8) is given by

$$\begin{aligned} u_{tt}(t) + aAu(t) - bA^\beta\theta(t) &= 0, \\ \theta_t(t) + B_1 q(t) + dA^\beta u_t(t) &= 0, \\ \tau q_t(t) + q(t) + B_2\theta(t) &= 0, \end{aligned} \qquad (10)$$

with abstract operators B_1, B_2 satisfying

$$-B_1 B_2 = cA^\alpha. \qquad (11)$$

Here, $u, \theta : [0,\infty) \to \mathcal{H}$, and $q : [0,\infty) \to (\mathcal{H})^m$ for some $m \in \mathbb{N}$. The operator B_2 maps its domain in $\mathcal{H}$ into $(\mathcal{H})^m$,

$$B_2 : D(B_2) \subset \mathcal{H} \to (\mathcal{H})^m,$$

and

$$B_1 : D(B_1) \subset (\mathcal{H})^m \to \mathcal{H}.$$

The thermoelastic plate model (7) is contained choosing realizations of the divergence operator "$c\,\mathrm{div}$" for B_1, and of the gradient operator "∇" for B_2, and $m = n$ in $\mathbb{R}^n$.

The exponential stability given for the Fourier model (8) and described by $\mathcal{A}_{es}(\tau = 0)$ in Figure 1, is lost for the Cattaneo model in any point different from $(\beta, \alpha) = (\frac{1}{2}, 1)$.

Theorem 1 *The region of exponential stability given for the Fourier model by $\mathcal{A}_{es}(\tau = 0)$ in (9) resp. Figure 1, is lost for the system (10) in any point different from the singular point $(\beta, \alpha) = (\frac{1}{2}, 1)$ which corresponds to (6).*

PROOF: The exponential stability of system (10) for $(\beta, \alpha) = (\frac{1}{2}, 1)$ (and $\tau > 0$ from now on) has been proved for the realization (6) for various boundary conditions [13].

We shall prove the *non*-exponential stability for the remaining values of $(\beta, \alpha) \in \mathcal{A}_{es}(\tau = 0)$. As ingredients of the proof we have an eigenfunction expansion and the Hurwitz criterion. For the proof we may assume for simplicity, but w.l.o.g., that the constants a, b, c, d appearing in (10), (11) satisfy $a = b = c = d = 1$.

The idea is to use the following ansatz of separation of variables via the eigenfunctions $(\phi_j)_j$ of A,

$$u_j(t) = a_j(t)\phi_j,\ \theta_j(t) = b_j(t)\phi_j,\ q_j(t) = c_j(t)B_2\phi_j, \quad (12)$$

for arbitrary j (assuming $B_2\phi_j$ not being identically zero), and to find solutions with decay contradicting exponential stability.

Using (11) we observe

$$B_1 q_j(t) = c_j(t)B_1B_2\phi_j = -c_j(t)A^\alpha\phi_j = -\lambda^\alpha c_j(t)\phi_j,$$

thus solving (10) is equivalent to solving the following system of ODEs for the coefficient functions (a_j, b_j, c_j), where a prime $'$ denotes differentiation with respect to time t,

$$\begin{array}{rcl} a_j'' + \lambda_j a_j - \lambda_j^\beta b_j &=& 0,\\ b_j' - \lambda_j^\alpha c_j + \lambda_j^\beta a_j' &=& 0,\\ \tau c_j' + c_j + b_j &=& 0. \end{array} \quad (13)$$

The last equation arises from the last equation in (10) with the ansatz (12) using again the natural condition that $B_2\phi_j$ is not identically zero.

System (13) is equivalent to a first-order system for the column vector $V_j := (a_j, a_j', b_j, q_j)$,

$$V_j' = \begin{pmatrix} 0 & 1 & 0 & 0\\ -\lambda_j & 0 & \lambda_j^\beta & 0\\ 0 & -\lambda_j^\beta & 0 & \lambda_j^\alpha\\ 0 & 0 & -\frac{1}{\tau} & -\frac{1}{\tau} \end{pmatrix} V_j \equiv A_j V_j. \quad (14)$$

We are looking for solutions to (14) of type

$$V_j(t) = \mathrm{e}^{\omega_j t} V_j^0.$$

In other words, ω_j has to be an eigenvalue of A_j with eigenvector V_j^0 as initial data.

It is the aim to demonstrate that, for any given small $\varepsilon > 0$, we have some j and some eigenvalue ω_j such that the real part $\Re\omega_j$ of ω_j is larger than $-\varepsilon$. This will contradict the exponential stability (being a kind of uniform property over all initial values), observing

$$|V_j(t)| = \mathrm{e}^{\Re w_j}|V_j^0|.$$

Computing the characteristic polynomial of A_j we have

$$\begin{array}{rcl} \det(A_j - \omega) &=& \frac{1}{\tau}\Big(\tau\omega^4 + \omega^3 + \left[\lambda_j^\alpha + \tau(\lambda_j + \lambda_j^{2\beta})\right]\omega^2 \\ && +[\lambda_j + \lambda_j^{2\beta}]\omega + \lambda_j^{1+\alpha}\Big)\\ &\equiv& \frac{1}{\tau}P_j(\omega). \end{array}$$

To reach our aim, i.e. to show that

$$\forall\, \varepsilon > 0\ \ \exists j\ \ \exists \omega_j,\ P_j(\omega_j) = 0:\ \ \Re\omega_j \geq -\varepsilon,$$

we introduce, for small $\varepsilon > 0$,

$$z := \omega + \varepsilon, \qquad P_{j,\varepsilon} := P_j(z - \varepsilon).$$

That is, we have to show

$$\forall\ 0 < \varepsilon \ll 1\ \ \exists j\ \ \exists z_j,\ P_{j,\varepsilon}(z_j) = 0:\ \ \Re z_j \geq 0. \quad (15)$$

To prove (15) we start with computing

$$P_{j,\varepsilon} = q_4z^4 + q_3z^3 + q_2z^2 + q_1z + q_0$$

where

$$\begin{array}{c} q_4 = \tau,\\ q_3 = -4\tau\varepsilon + 1,\\ q_2 = 6\tau\varepsilon^2 - 3\varepsilon + \lambda_j^\alpha + \tau(\lambda_j + \lambda_j^{2\beta}),\\ q_1 = -4\tau\varepsilon^3 + 3\varepsilon^2 - 2\left(\lambda_j^\alpha + \tau(\lambda_j + \lambda_j^{2\beta})\right)\varepsilon\\ +\lambda_j + \lambda_j^{2\beta},\\ q_0 = \tau\varepsilon^4 - \varepsilon^3 + \left(\lambda_j^\alpha + \tau(\lambda_j + \lambda_j^{2\beta})\right)\varepsilon^2\\ -(\lambda_j + \lambda_j^{2\beta})\varepsilon + \lambda_j^{1+\alpha}. \end{array}$$

Since $\lambda_j \geq \lambda_1 > 0$, there is $0 < \varepsilon_0 < \frac{1}{4\tau}$ such that for all $0 < \varepsilon \leq \varepsilon_0$ the coefficients $q_4, \ldots, q_0$ are positive. So we assume w.l.o.g. from now on that $0 < \varepsilon \leq \varepsilon_0 < \frac{1}{4\tau}$.

We use the Hurwitz criterion [14]: Let

$$\mathbb{H}^j := \begin{pmatrix} q_3 & q_4 & 0 & 0\\ q_1 & q_2 & q_3 & q_4\\ 0 & q_0 & q_1 & q_2\\ 0 & 0 & 0 & q_0 \end{pmatrix}$$

denote the Hurwitz matrix associated to the polynomial $P_{j,\varepsilon}$. Then (15) is fulfilled if we find, for given small $\varepsilon > 0$, a (sufficiently large) index j such that one of the principal minors of $\mathbb{H}^j$ is not positive. The principal minors are given by the determinants $\det D_m^j$ of the matrices D_m^j, for $m = 1, 2, 3, 4$, where D_m^j denotes the upper left square submatrix of $\mathbb{H}^j$ consisting of the elements $\mathbb{H}_{11}^j, \ldots, \mathbb{H}_{mm}^j$.

Since

$$\det D_1^j = q_3 > 0 \quad \text{and} \quad \det D_4^j = q_0 \det D_3^j,$$

with positive q_0, it remains to prove that

$$\text{either}\ \ \det D_2^j \leq 0 \quad \text{or} \ \ \det D_3^j \leq 0,$$

for some (sufficiently large) j.

The set of parameters (β, α) for which we have to prove this, will be divided into two subsets, the first one, where $\alpha < 1$, and the second one, where $\alpha = 1$ and $\frac{1}{2} < \beta \leq 1$:

Part I: $\alpha < 1$.

We have

$$\begin{array}{rcl} \det D_2^j &=& q_3q_2 - q_4q_1\\ &=& [1 - 4\tau\varepsilon]\cdot\\ && \cdot[6\tau\varepsilon^2 - 3\varepsilon + \lambda_j^\alpha + \tau(\lambda_j + \lambda_j^{2\beta})] -\\ && \tau[-4\tau\varepsilon^3 + 3\varepsilon^2 - 2(\lambda_j^\alpha + \tau(\lambda_j + \lambda_j^{2\beta}))\varepsilon +\\ && \lambda_j + \lambda_j^{2\beta}]\\ &=& -2\tau^2\varepsilon\lambda_j^{2\beta} - 2\tau^2\varepsilon\lambda_j + (1 - 2\tau\varepsilon)\lambda_j^\alpha -\\ && 20\tau^2\varepsilon^3 + 15\tau\varepsilon^2 - 3\varepsilon \end{array}$$

implying

$$\det D_2^j \leq -2\tau^2\varepsilon\lambda_j + \mathcal{O}(\lambda_j^\alpha), \tag{16}$$

where we use the Landau symbol $\mathcal{O}(\lambda_j^\alpha)$ to denote a term satisfying

$$|\mathcal{O}(\lambda_j^\alpha))| \leq k_1\lambda_j^\alpha$$

with a positive constant k_1 (being independent of j, ε, τ). Thus we conclude from (16)

$$\det D_2^j < 0 \tag{17}$$

for sufficiently large j (depending on ε, τ) since $\alpha < 1$ and $\lambda_j \to \infty$ by assumption.

Part II: $\alpha = 1$, $\frac{1}{2} < \beta \leq 1$.

We compute

$$\begin{aligned}
\det D_3^j &= q_1 \det D_2^j - q_3^2 q_0 \\
&= [(1-2\tau\varepsilon)\lambda_j^{2\beta} + \mathcal{O}(\lambda_j)] \cdot \\
&\quad \cdot[-2\tau^2\varepsilon\lambda_j^{2\beta} + \mathcal{O}(\lambda_j)] - [(1-4\tau\varepsilon)^2] \cdot \\
&\quad \cdot[\lambda_j^2 + \varepsilon(\tau\varepsilon - 1)\lambda_j^{2\beta} + \mathcal{O}(\lambda_j)] \\
&= -2\tau^2\varepsilon(1-2\tau\varepsilon)\lambda_j^{4\beta} + \mathcal{O}(\lambda_j^{2\beta+1}) \\
&\leq -\tau^2\varepsilon\lambda_j^{4\beta} + \mathcal{O}(\lambda_j^{2\beta+1}).
\end{aligned}$$

implying

$$\det D_3^j < 0 \tag{18}$$

for sufficiently large j (depending on ε, τ) since $\beta > \frac{1}{2}$ implies $2 < 2\beta + 1 < 4\beta$, and since $\lambda_j \to \infty$.

With (17) and (18) we have proved (15) and hence Theorem 1. □

CONCLUSION

In modeling heat conduction, different models using Fourier's law on one hand, or using Cattaneo's law on the other hand, can lead to very similar results concerning the asymptotic behavior in time of solutions. This holds for pure heat equations (4), (5), or for thermoelasticity of second order (6).

Examples of thermoelastic plates (7) or for Timoshenko beams exhibit a different behavior: exponential stability under the Fourier law, but no exponential stability under the Cattaneo law.

We have shown that the behavior in these two special examples is typical for a large class of coupled thermoelastic systems given as abstract model in (10). This system (10) includes the models for thermoelastic plates (7), for thermoelasticity of second order (6), and for more, like viscoelastic equations.

As a result, for all models, i.e. values of the parameters (β, α), the property of exponential stability given under the Fourier law is lost under the Cattaneo law for all parameter values which are different from the pair representing thermoelasticity of second order.

As a consequence, the considerations above should trigger a discussion of the "right" modeling in heat conduction among scientists working in modeling, in fundamental analysis, and in implementations of these models in applications.

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