Universität Konstanz
Fachbereich Mathematik und Statistik
C. Scheiderer, Th. Mayer
SS 2020

Universität Konstanz

Exercises for Real Algebraic Geometry II

Sheet 5

Please upload your solutions on ILIAS by Monday 25 May 2020 at 11:45

Exercise 17
Let $n \in \mathbb{N}$, let k be a field and $A \in \mathrm{M}_n(k)$ an $n \times n$-matrix over k with $\det(A) = 0$. Show that A is a nonsingular point of the hypersurface $\det = 0$ (in affine n^2-space) if and only if $\mathrm{rk}(A) = n - 1$.

Exercise 18
Let X be the plane affine curve $y^3 + 2x^2y - x^4 = 0$ over $\mathbb{R}$. Show that X is irreducible, find the singular $\mathbb{C}$-points of X and show that $X(\mathbb{R})$ is a 1-dimensional differentiable submanifold of $\mathbb{R}^2$.

Exercise 19
State and prove a version of the Jacobian criterion for nonsingular points on projective varieties.

Exercise 20
Find the complex singular points of the plane curve

$$(x^2 + y^2 - z^2)^3 \;=\; x^2y^3z.$$