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Exercises for Real Algebraic Geometry II

Sheet 6

Please upload your solutions on ILIAS by Monday 1 June 2020 at 11:45

Exercise 21

Find an example of a field k and an irreducible affine k-variety X such that the ring $k[X] \otimes_k \overline{k}$ is not reduced.

Exercise 22

Let $(A, \mathfrak{m}, k)$ be a henselian local ring (with $\frac{1}{2} \in A$). Then every closed point α of $\operatorname{Sper}(A)$ satisfies $\operatorname{supp}(\alpha) = \mathfrak{m}$. Show by an example that this usually fails without the henselian hypothesis.

Extra challenge (voluntary): Show that the first assertion remains true even without the assumption $\frac{1}{2} \in A$.

Exercise 23

Let k be a field, let $A = k[\![x_1, \dots, x_n]\!]$ be the ring of formal power series. Given $f = \sum_{i \geq 0} a_i y^i$ in $A[\![y]\!]$ (with $a_i \in A$), let

$$v(f) := \min\{\omega(a_i)\colon\ i \geq 0\}$$

($f \neq 0$), and put $v(0) := \infty$. Show that $v(fg) = v(f) + v(g)$ holds for all f, $g \in A[\![y]\!]$.

Exercise 24

Let A be a discrete valuation ring, let $K = \operatorname{Quot}(A)$, and let v be the discrete valuation of K corresponding to A.

(a) Let $g = \sum_{n \geq 0} a_n t^n \in A[\![t]\!]$ be a power series with $a_0 \neq 0$, and let $g^{-1} = \sum_{n \geq 0} b_n t^n$ in $K[\![t]\!]$. Then $v(b_n) \geq -(n+1)v(a_0)$ for every $n \geq 0$.

(b) If $f = \sum_{n \geq n_0} c_n t^n \in K(\!(t)\!)$ (with $c_n \in K$), and if $\liminf_{n \to \infty} \frac{v(c_n)}{n} = -\infty$, then $f \notin \operatorname{Quot}(A[\![t]\!])$.

(c) Let k be a field. Find an explicit element of $k(\!(x_1)\!)(\!(x_2)\!)$ that does not lie in $k(\!(x_1, x_2)\!)$.