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Exercises for Real Algebraic Geometry II

Sheet 7

Please upload your solutions on ILIAS by Monday 15 June 2020 at 11:45

Exercise 25

Let k be a field, and let $K = k((x))$. Find all finite field extensions of K up to K-isomorphism when k is (a) algebraically closed of characteristic zero, (b) real closed.

Exercise 26

Let k be a field with $\text{char}(k) = p > 0$, let $F = k((x^{1/\infty}))$ be the field of formal Puiseux series over k.

(a) There exists $f \in F$ with $f^p + f = x$. Find such f.
(b) Prove that there doesn't exist $f \in F$ with $f^p + f = x^{-1}$. In particular, the field F is not algebraically closed.

Exercise 27

Let R be a real closed field, let $a, b \in R$ with $|a|, |b| \leq 1$. In the polynomial ring $R[x]$ show that $(x-a)(x-b) \in PO(x^2-1)$.

Exercise 28

Let A be a ring, and let $T = PO_A(g_i \colon i \in I)$ where $g_i \in A$ $(i \in I)$ is a family of elements. For the ring $B = A[x_i \colon i \in I]/\langle x_i^2 - g_i \colon i \in I\rangle$ and for every $f \in A$ show that:

(a) $f \in T$ if and only if f is sos in B;
(b) $f \in \text{Sat}(T)$ if and only if f is psd in B.