Universität Konstanz
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Universität Konstanz

Exercises for Real Algebraic Geometry II

Sheet 8

Please upload your solutions on ILIAS by Monday 22 June 2020 at 11:45

Exercise 29

Let $g(x) \in \mathbb{R}[x]$ be a polynomial of odd degree ≥ 3, and let C be the plane affine curve $y^2 = g(x)$. Then psd $\neq$ sos holds in $\mathbb{R}[C]$.

Exercise 30

Let A be a semilocal ring, and let α, $\beta \in \mathrm{Sper}(A)$ be such that $\mathrm{sign}_\alpha(u) = \mathrm{sign}_\beta(u)$ for every unit u of A. Then α and β have a common specialization in $\mathrm{Sper}(A)$. (*Hint*: Prove it indirectly, using Winter semester, Aufgabe 40.)

Exercise 31

Let $B = \mathbb{R}[x, y]$ and $f = x$, $g = y^2 + 1 - x \in B$, and let $\mathfrak{m} = \langle x, y^2 + 1\rangle \subseteq B$. The local ring $A := B_{\mathfrak{m}}/\langle fg\rangle$ has Krull dimension one. Show that

(a) f is psd in A,
(b) f is sos in $\widehat{A}$,
(c) f is not sos in A.

Exercise 32

Let k be a real field. Show that psd = sos does not hold in the local ring $A = k[\![x, y]\!]/\langle y^2 - x^3\rangle$.