

EXACTNESS OF THE PPT CONDITION FOR 3X3 MATRICES

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ABSTRACT. We consider a family $P_{n,m}$ of cones of non-negative polynomials and the corresponding cones $\Sigma_{n,m}$ of sums of squares. The cone $P_{n,m}$ can be described as the set of those linear mappings from the space \mathbf{R}^n into the space of real symmetric $m \times m$ matrices which map the n -dimensional Lorentz cone into the cone of real symmetric positive semidefinite matrices. Our main result is that $P_{n,m} = \Sigma_{n,m}$ for $m = 3$. Sums of squares relaxations for optimisation problems over the cones $P_{n,3}$ thus yield the exact result. In particular, the matrix ellipsoid problem for real symmetric 3×3 matrices can be written as a Linear Matrix Inequality. We provide an example of a polynomial in $P_{n,m} \setminus \Sigma_{n,m}$ for $m \geq 4, n \geq 4$. Hence $P_{n,m} = \Sigma_{n,m}$ if and only if $\min(n, m) \leq 3$.