Konstanz, August 26, 2010

Cordian Riener nominated a postdoctoral fellowships in the Zukunftskolleg

On behalf of the Research Initiative "Real Algebraic Geometry and Emerging Applications" Cordian Riener has been nominated for a postdoctoral fellowship at the Zukunftskolleg for the period from April 2011 to October 2012. This nomination has been approved by the Zukunftskolleg on June 22, 2010.

The Zukunftskolleg is a central scientific institution of the University of Konstanz for the promotion of young scientists in the natural sciences, humanities, and social sciences. It aims to recruit young researchers who have demonstrated outstanding academic achievement and whose projects show evidence of unique potential. They are associate members of the Zukunftskolleg and participate in the Zukunftskolleg's weekly jour fixe meetings. Associate membership entitles fellows to attend Zukunftskolleg events and take advantage of the services provided by the Central Office of the Zukunftskolleg.

The Research Initiative "Real Algebraic Geometry and Emerging Applications" funded by the Excellence Initiative will start on October 1, 2010. It is centered within the research focus "Real Geometry and Algebra" of the Department of Mathematics and Statistics with a strong branch in Leipzig and ties to Frankfurt and Magdeburg. This research focus is currently in its constitutional phase. It is built around professors Claus Scheiderer being here since 2004, Salma Kuhlmann and myself being here since last winter term, and a currently recruited junior professor to come in autumn.

One of the topics of Cordian Riener's thesis is Timofte's theorem. Timofte's theorem is a seminal result saying in its simplest form that a symmetric polynomial (a polynomial invariant under the full symmetric group acting on its variables) of degree $d \geq 4$ in several variables is positive if and only if it is positive on all points with at most $d/2$ different components. Though the result is purely algebraic, Timofte's proof is very analytical and uses amongst others the theory of ordinary differential equations. From the theory of real closed fields and Gödel's completeness theorem, I knew that for each fixed degree d, there must exist an algebraic proof of this result. I thus thought that it should most likely be possible to give an algebraic proof which works even uniformly in each degree d.

Cordian Riener could give a very surprising reformulation of Timofte's theorem in terms of hyperbolic polynomials in one variable. In our discussions, we found a connection to old algebraic theorems of the Bulgarian mathematician Nikola Dimitrov Obreschkoff. In the sequel, however, Cordian Riener eliminated even these ingredients and eventually managed to give a beautiful and comprehensive algebraic proof of Timofte's result using techniques from both Optimization and Real Algebra. Cordian Riener's proof deepens the understanding of Timofte's result and makes it more accessible especially for people working in algebra.

A polynomial optimization problem is the problem of minimizing or maximizing a polynomial objective function subject to constraints given by polynomial inequalities. Since the beginning of the millennium, the theory of solving such problems evolved dramatically with the introduction of Lasserre's relaxation method. This method relates polynomial optimization to one of the core subjects of Real Algebraic Geometry (sums of squares representations of real polynomials) and to its dual theme from Functional Analysis (the truncated moment problem). A good introduction to this method is provided by the survey "Positive polynomials and semidefinite programming" (Jahresbericht der Deutschen Mathematiker-Vereinigung 110, 2008) written by Cordian Riener and Thorsten Theobald.

While Lasserre's relaxation scheme is very successfully applied to many practical cases of polynomial optimization problems, it will however be too slow to solve the general case, simply because the considered problems are too hard (even unconstrained polynomial optimization with objective functions of degree four is already NP-hard). The interesting case in practice and in theory, is therefore to add additional assumptions present in many real world applications and possibly yielding to polynomial time algorithms. Such assumptions are *symmetry* and *sparsity*. Part of Cordian Riener's future plans is to get stronger results in the theory of polynomial optimization under such additional structure.

Timofte's result treats only symmetry under the natural action of the full symmetric group. Unlike Timofte's proof, it seems that Cordian Riener's approach can be further extended and generalized to actions of other groups showing that Timofte's result could be the precursor of a whole new theory. The mathematical machinery of *Invariant Theory* is very well developed and seems now ready to be applied due to Cordian Riener's work.

I am convinced that a theory of polynomial optimization tailored toward real world problems needs all three ingredients, Optimization, Invariant Theory and Real Algebraic Geometry. It would be an excellent opportunity to broaden the scope of our Research Initiative if Cordian Riener could complement it by bringing in the additional aspect of symmetries and sparsity. This would certainly make our forthcoming application for a Research Unit more competitive. For Cordian Riener on the other hand, it is now the time to get into a more algebraic environment so that he can apply the machinery of Real Algebraic Geometry and Algebraic Invariant Theory to Polynomial Optimization.

Prof. Dr. Markus Schweighofer