



Compact Seminar on Profinite Groups

Period: March 6th to 31st, two talks per week for four weeks.

Main Reference: L. Ribes and P. Zalesskii, *Profinite Groups*, Springer-Verlag 2010, ISBN: 978-3-642-01641-7

Here is a list of topics that we aim at covering, over the eight talks. References are to the Main Reference above.

Week 1 - March 6-12

1 Introduction I (Michele)

Chapter 1, pag. 1-12.

Topological groups (definition and examples)

Projective Systems

- Existence and uniqueness of the projective limit
- X_i Hausdorff $\Rightarrow \varprojlim X_i \leq_c \prod X_i$
- Further properties
- Morphisms of Projective Systems

Profinite spaces and groups

- Characterisation of profinite spaces (resp. groups) (Theorem 1.1.12)
- Examples (express $\prod X_i$ as a projective limit)

2 Introduction II (Michele)

Chapter 2, pag. 19-28.

Classes (\mathcal{C}) of finite groups

- \mathcal{C} can be the class of finite groups, finite p -groups, abelian groups, cyclic groups.

Pro- \mathcal{C} -groups

- Properties a family \mathcal{C} can satisfy (e.g., closed under taking subgroups, quotients) and examples.
- Fundamental system of open neighbourhoods of 1 of a profinite group (Lemma 2.1.1)
- Equivalent definitions of pro- \mathcal{C} -group (Theorem 2.1.3)
- Closed subgroups
- Completions (Examples: $\hat{\mathbb{Z}}, \mathbb{Z}_p$)

Week 2 - March 13-19

3 Basic properties of profinite groups (Marco)

Chapter 2, sections 2.2, 2.3, pag. 28-32.

- Properties of \mathcal{C} inherited by pro- \mathcal{C} -groups (Prop. 2.2.1)
- Order of a profinite group (Prop. 2.3.1)
- Existence of sections (Prop. 2.2.2)
- Examples (end of pag. 29, Exercise 2.2.3)

4 Sylow theorems for profinite groups (Zhelun)

Chapter 2, section 2.3, pag. 33-38.

- Supernatural numbers, π -Hall groups and p -Sylow groups
- The Sylow theorems (Lemma 2.3.4, Theorem 2.3.5, Corollary 2.3.6)

Week 3 - March 20-26

5 Generators (Michael)

Chapter 2, sections 2.4 and 2.5, pag. 43-48.

- Sets of (topological) generators
- Existence of a set of generators converging to 1 (Prop. 2.4.2)
- Finitely generated profinite groups (Prop. 2.5.1, Prop. 2.5.5)

6 The Frattini subgroup of a profinite group (Sebastian)

Chapter 2, pag. 54-58

- Section 2.8 up to Prop. 2.8.10

Week 4 - March 27-31

7 Profinite groups as Galois groups (Pia)

Chapter 2, section 2.11, pag. 70-74

- Reminders of Galois correspondence
- The Krull topology
- Galois groups, endowed with the Krull topology are profinite (Theorem 2.11.1)
- The fundamental theorem for infinite Galois extensions (Theorem 2.11.3)
- Inverse Galois problem for profinite groups (Theorem 2.11.5)

8 Procylic Groups (Fabian)

Chapter 2, section 2.3, pag. 39-40 and section 2.7, pag. 52-53

- Classification by order (Prop. 2.7.1)
- A procylic group is the product of its p -Sylows (Prop. 2.3.8)
- Theorem 2.7.2 (description of general procylic groups, immediate consequence of previous results)

- The p -adic integers \mathbb{Z}_p and the Prüfer group $\hat{\mathbb{Z}}$ (Prop. 2.3.9, 2.3.10, Examples 2.3.11, 2.4.2)