Universität Konstanz
Fachbereich Mathematik und Statistik
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Wintersemester 2019/2020

Exercises for the course "Linear Algebra I"

Sheet 2

Hand in your solutions on Thursday, 7. November 2019, 09:55, in the Postbox of your Tutor in F4. Please try to write your solutions clear and readable, write your name and the name of your tutor on each sheet, and staple the sheets together.

Aufgabe 2.1 (*4 Punkte*)

Using the euclidean algorithm and its reversed procedure, determine:

(a) $\gcd(193, 60)$ (gcd = ggT; "greatest common divisor")
(b) Two integer numbers $x, y \in \mathbb{Z}$ such that $169x + 144y = 1$
(c) The multiplicative inverse of 4 in $\mathbb{Z}_{19}$
(d) The multiplicative inverse of 10 in $\mathbb{Z}_{27}$

Please write all the steps to your solution!

Aufgabe 2.2 (*4 Punkte*)

For $n = 5, 6$

(a) Write the Cayley table (Verknüpfungstafeln) for the addition and multiplication in $\mathbb{Z}_n$;
(b) determine the multiplicatively invertible elements in $\mathbb{Z}_n$.

Aufgabe 2.3 (*4 Punkte*)

For each of the following values of n find all elements $x \in \mathbb{Z}_n$ that satisfy the corresponding equation. (Beware that the set of solutions may be empty!)

(a) $n = 5$, $3 \cdot_5 x = 1$
(b) $n = 11$, $x^2 +_{11} 1 = 0$ (where $x^2 = x \cdot_{11} x$)
(c) $n = 9$, $x^3 = 0$ (where $x^3 = x \cdot_9 x \cdot_9 x$)
(d) $n = 12$, $2 \cdot_{12} x = 3$

Aufgabe 2.4 (*4 Punkte*)

Let $+_n$ and $\cdot_n$ be the operations defined in class. It was already showed in the lecture that $(\mathbb{Z}_n, +_n)$ is an abelian group.

The goal of this exercise is to show that $(\mathbb{Z}_n, +_n, \cdot_n)$ is a commutative ring with one. Show that:

(a) The operation $\cdot_n$ is associative.
(b) The operation $\cdot_n$ is commutative.
(c) 1 is a neutral element with respect to $\cdot_n$.
($\star$) The remaining fact, that $\cdot_n$ is distributive with respect to $+_n$, will be proved in the Plenumsübung on November 12th so you don't need to prove it here. Nevertheless, we strongly recommend that you try to write a proof by yourselves before seeing in in the Plenumsübung.