Theory Graphs and Meta-Logical/Grammatical Frameworks: MMT as a Logic/Language/World-Workbench

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1 Introduction & Motivation
About Humans and Computers in Mathematics

- Computers and Humans have complementary strengths.
  - Computers can handle large data and computations flawlessly at enormous speeds.
  - Humans can sense the environment, react to unforeseen circumstances and use their intuitions to guide them through only partially understood situations.

In mathematics: we exploit this, we
- let humans explore mathematical theories and come up with novel insights/proofs,
- delegate symbolic/numeric computation and typesetting of documents to computers.
- (sometimes) delegate proof checking and search for trivial proofs to computers.

Overlooked Opportunity: management of existing mathematical knowledge
- cataloguing, retrieval, refactoring, plausibilization, change propagation and in some cases even application do not require (human) insights and intuition
- can even be automated in the near future given suitable representation formats and algorithms.

Math. Knowledge Management (MKM): is the discipline that studies this.

Application: Scaling Math beyond the One-Brain-Banner
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**The One-Brain-Barrier**

- **Observation 1.1.** More than $10^5$ math articles published annually in Math.
- **Observation 1.2.** The libraries of Mizar, Coq, Isabelle, ... have $\sim 10^5$ statements + proofs each. (but are mutually incompatible)
- **Consequence:** humans lack overview over – let alone working knowledge in – all of math/formalizations. (Leonardo da Vinci was said to be the last who had)
- **Dire Consequences:** duplication of work and missed opportunities for the application of mathematical/formal results.
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▶ **Problem:** Math Information systems like arXiv.org, Zentralblatt Math, MathSciNet, etc. do not help (only make documents available)

▶ **Fundamenal Problem:** the One-Brain Barrier (OBB)
  ▶ To become productive, math must pass through a brain
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- **Fundamenal Problem:** the One-Brain Barrier (OBB)
  - To become productive, math must pass through a brain
  - Human brains have limited capacity (compared to knowledge available online)
- **Idea:** enlist computers (large is what they are good at)
- **Prerequisite:** make math knowledge machine-actionable & foundation-independent (use MKM)
2 Modular Representation of Mathematics
Idea: Follow mathematical practice of generalizing and framing

- framing: If we can view an object \( a \) as an instance of concept \( B \), we can inherit all of \( B \) properties (almost for free.)
- state all assertions about properties as general as possible (to maximize inheritance)
- examples and applications are just special framings.

Modern expositions of Mathematics follow this rule (radically e.g. in Bourbaki)

formalized in the theory graph paradigm (little/tiny theory doctrine)

- theories as collections of symbol declarations and axioms (model assumptions)
- theory morphisms as mappings that translate axioms into theorems

Example 2.1 (MMT: Modular Mathematical Theories). MMT is a foundation-indenpent theory graph formalism with advanced theory morphisms.

Problem: With a proliferation of abstract (tiny) theories readability and accessibility suffers (one reason why the Bourbaki books fell out of favor)
Modular Representation of Math (MMT Example)

\[ \text{IntArith} \]
\[ Z := p/N \cup n/N \]
\[ -0=0 \]

\[ \text{NatTimes} \]
\[ n \cdot 1=n, \quad n \cdot s(m)=n \cdot m+n \]

\[ \text{NatPlus} \]
\[ n+0=n, \quad n+s(m)=s(n+m) \]

\[ \text{NatNums} \]
\[ \mathbb{N}, \ s, \ 0 \]
\[ P1, \ldots P5 \]

\[ \psi = \{ \ m \mapsto e, \ a \mapsto c \} \]

\[ \varphi = \{ \ G \mapsto \mathbb{N}, \ o \mapsto \cdot, \ e \mapsto 1 \} \]

\[ \psi = \{ \ G \mapsto \mathbb{N}, \ o \mapsto +, \ e \mapsto 0 \} \]

\[ \psi' = \{ \ i \mapsto -, \ g \mapsto f \} \]

\[ \text{CGroup} \]
\[ \text{comm}: \text{xoy}=\text{yox} \]

\[ \text{Group} \]
\[ \text{i} := \lambda x.\tau y.\text{xoy}=e \]
\[ \forall x:G.\exists y:G.\text{xoy}=e \]

\[ \text{NonGrpMon} \]
\[ \exists x:G.\forall y:G.\text{xoy}\neq e \]

\[ \text{Ring} \]
\[ x \cdot m/0 \ (y a/0 z)=(x \cdot m/y) \ a/0 (x \cdot m/0 z) \]

\[ \text{Monoid} \]
\[ e \cdot \text{ox}=x \]

\[ \text{SemiGrp} \]
\[ \text{assoc}: (\text{xoy})oz=\text{x0(yoz)} \]

\[ \text{Magma} \]
\[ \text{G, o} \]
\[ \text{xoy}\in G \]

\[ \{ x \circ y \mapsto y \circ x \} \]

\[ \{ x \circ y \mapsto y \circ x \} \]
Concrete MMT Syntax

Example 2.2 (A Theory and Type for Unital Magmas).

```plaintext
theory Unital : base:?Logic =
  include ?Magma |

theory unital_theory : base:?Logic =
  include ?Magma/magma_theory |
  unit : U | # e prec -1 |
  axiom_leftUnital : ⊢ prop_leftUnital op e |
  axiom_rightUnital : ⊢ prop_rightUnital op e |

unital = Mod unital_theory |

unit0f : \{G: unital\} dom G | # `II e prec 5 | = [G] (G.unit) |
```

where the following is imported with ?Magma

```plaintext
prop_leftUnital : \{U : type\} (U → U → U) → U → prop |
  = [U,op,e] ∀[x] op e x ≡ x | # prop_leftUnital 2 3 |

prop_rightUnital : \{U : type\} (U → U → U) → U → prop |
  = [U,op,e] ∀[x] op x e ≡ x | # prop_rightUnital 2 3 |
```
The MMT Module System

- **Central notion**: theory graph with theory nodes and theory morphisms as edges

- **Definition 2.3.** In MMT, a **theory** is a sequence of constant declarations – optionally with type declarations and definitions

- MMT employs the Curry/Howard isomorphism and treats
  - axioms/conjectures as typed symbol declarations (propositions-as-types)
  - inference rules as function types (proof transformers)
  - theorems as definitions (proof terms for conjectures)

- **Definition 2.4.** MMT had two kinds of theory morphisms
  - **structures** instantiate theories in a new context (also called: definitional link, import)
    - they import of theory $S$ into theory $T$ induces theory morphism $S \to T$
  - **views** translate between existing theories (also called: postulated link, theorem link)
    - views transport theorems from source to target (framing)

- together, structures and views allow a very high degree of re-use

- **Definition 2.5.** We call a statement $t$ **induced** in a theory $T$, iff there is
  - a path of theory morphisms from a theory $S$ to $T$ with (joint) assignment $\sigma$,
  - such that $t = \sigma(s)$ for some statement $s$ in $S$.

- In MMT, all induced statements have a canonical name, the MMT URI.
search on the LATIN Logic Atlas

Flattening the LATIN Atlas (once):

<table>
<thead>
<tr>
<th>type</th>
<th>modular</th>
<th>flat</th>
<th>factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>declarations</td>
<td>2310</td>
<td>58847</td>
<td>25.4</td>
</tr>
<tr>
<td>library size</td>
<td>23.9 MB</td>
<td>1.8 GB</td>
<td>14.8</td>
</tr>
<tr>
<td>math sub-library</td>
<td>2.3 MB</td>
<td>79 MB</td>
<td>34.3</td>
</tr>
<tr>
<td>MathWebSearch harvests</td>
<td>25.2 MB</td>
<td>539.0 MB</td>
<td>21.3</td>
</tr>
</tbody>
</table>

simple search frontend at http://cds.omdoc.org:8181/search.html

FlatSearch DEMO

\[ X + Y \]


assoc: == (+ (+ X Y) Z) (+ X (+ Y Z))

Justification

Induced statement found in http://latin.omdoc.org/math?IntAryth
IntAryth is a AbelianGroup if we interpret over view g
AbelianGroup contains the statement assoc

Applications for Theories in Physics

- Theory Morphisms allow to “view” source theory in terms of target theory.
- Theory Morphisms occur in Physics all the time.

<table>
<thead>
<tr>
<th>Theory</th>
<th>Temp. in Kelvin</th>
<th>Temp. in Celsius</th>
<th>Temp. in Fahrenheit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signature</td>
<td>°K</td>
<td>°C</td>
<td>°F</td>
</tr>
<tr>
<td>Axiom:</td>
<td>absolute zero at 0°K</td>
<td>Water freezes at 0°C</td>
<td>cold winter night: 0°F</td>
</tr>
<tr>
<td>Axiom:</td>
<td>δ(°K1) = δ(°C1)</td>
<td>Water boils at 100°C</td>
<td>domestic pig: 100°F</td>
</tr>
<tr>
<td>Theorem:</td>
<td>Water freezes at 271.3°K</td>
<td>domestic pig: 38°C</td>
<td>Water boils at 170°F</td>
</tr>
<tr>
<td>Theorem:</td>
<td>cold winter night: 240°K</td>
<td>absolute zero at −271.3°C</td>
<td>absolute zero at −460°F</td>
</tr>
</tbody>
</table>

Views: °C ↦+271.3° K, °C ↦−32/2° F, and °F ↦+240/2° K, inverses.

- Other Examples: Coordinate Transformations,
- Application: Unit Conversion: apply view morphism (flatten) and simplify with UOM.
  (For new units, just add theories and views.)
- Application: MathWebSearch on flattened theory (Explain view path)
3 Foundational Pluralism (the Meta-Meta Level)
Assembling a Global Knowledge Resource (Problems)

- **Problems**: encountered in practice
  - Different systems have different, mutually incompatible logical/mathematical foundations (hundreds, optimize different aspects)
  - the respective communities are largely disjoint
  - have built large, incompatible, but mathematically overlapping libraries
  - all tools lack crucial features (cannot afford to develop)
  - new logics/foundations/systems seldom get off the ground (too expensive)

- **Definition 3.1.** A **foundation** (of mathematics) consists of
  - a foundational language (e.g. first-order logic or the calculus of constructions)
  - a foundational theory (e.g. axiomatic set theory)

**Observation**: need a system that can deal with multiple foundations $\sim$ foundational pluralism
Realizing Foundational Pluralism

Towards Integration at the Foundation Level:

<table>
<thead>
<tr>
<th>peer to peer</th>
<th>open standard</th>
<th>industry standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>F E G H A B D C</td>
<td>F E G H A B C D</td>
<td>F E G H A B C D</td>
</tr>
</tbody>
</table>

- **n^2 / 2 translations** symmetric
- **2n translations** symmetric
- **2n – 2 translations** asymmetric

**Problem:** So far So Obvious! But what should be in the middle?

**Idea (reused):** A modular representation of foundations (logics/theories) Bring-Your-Own-Foundation ~ foundation independent systems/tools

Kohlhase: MMT as a Logic/Language/*-Workbench 14. Jan. 19; Konstanz
Example 3.2. Logics and foundations represented as MMT theories

\[ \begin{array}{ccc}
\text{LF} & \rightarrow & \text{LF+X} \\
\downarrow & & \downarrow \\
\text{ZFC} & \xleftarrow{\text{folsem}} & \text{FOL} & \xrightarrow{f2h} & \text{HOL} \\
\downarrow & & \downarrow & \downarrow & \downarrow \\
\text{Monoid} & \xleftarrow{\text{mod}} & \text{CGroup} & \xrightarrow{\text{add}} & \text{Ring} \\
\end{array} \]

Definition 3.3. Meta-relation between theories – special case of inclusion

Uniform Meaning Space: morphisms between formalizations in different logics become possible via meta-morphisms.

Remark 3.4. Semantics of logics as views into foundations, e.g., folsem.

Remark 3.5. Models represented as views into foundations (e.g. ZFC)

Example 3.6. \( \text{mod} := \{ G \mapsto \mathbb{Z}, \circ \mapsto +, e \mapsto 0 \} \) interprets Monoid in ZFC.
**Definition 3.7.** The LATIN project (*Logic Atlas and Integrator*)

**Idea:** Provide a standardized, well-documented set of theories for logical languages, logic morphisms as theory morphisms.

**Technically:** Use MMT as a representation language *logics-as-theories*

Integrate logic-based software systems via views.

**State:** \(\sim\) 1000 modules (theories and morphisms) written in MMT/LF [RS09]
MMT a Module System for Mathematical Content

- **MMT**: Universal representation language for formal mathematical/logical content

- **Implementation**: MMT API with generic
  - module system for math libraries, logics, foundations
  - parsing + type reconstruction + simplification
  - IDEs (web server + JEdit+IntelliJ)
  - change management

- **Continuous development since 2007** (>$30000$ lines of Scala code)

- **Close relatives**:
  - LF, Isabelle, Dedukti: but flexible choice of logical framework
  - Hets: but declarative logic definitions
Example 3.8 (Propositional Logic (Syntax)).

```
theory PropLogSyntax : ur:?LF =
  prop : type | # bool |

  and  : bool → bool → bool | # 1 ∧ 2 prec 45 | /T jwedge |
  not  : bool → bool | # ¬ 1 prec 50 | /T jneg |

  or   : bool → bool → bool | # 1 ∨ 2 prec 40 |
  = [a,b] ¬ (¬ a ∧ ¬ b) | /T jvee |

  implies : bool → bool → bool | # 1 ⇒ 2 prec 35 |
  = [a,b] ¬ a v b | /T jra |

  iff : bool → bool → bool | # 1 ⇔ 2 prec 40 | = [a,b] (a ⇒ b) ∧ (b ⇒ a) |

  true : bool | # ⊤ | /T jtop |
  false : bool | = ¬ ⊤ | # ⊥ | /T jbot |
```
Example 3.9 (Propositional Logic (Natural Deduction)).

```
theory PropLogNatDed : ur:?LF =
  include ?PropLogSyntax |

  ded : bool → type | # ↑ 1 prec 1 | /T jvdash |

  andEl : {A,B} ⊢ A ∧ B → ⊢ A | # andEl 3 |
  andEr : {A,B} ⊢ A ∧ B → ⊢ B | # andEr 3 |
  andI : {A,B} ⊢ A → ⊢ B → ⊢ A ∧ B | # andI 3 4 |

  implI : {A,B} ( ⊢ A → ⊢ B ) → ⊢ A ⇒ B | # implI 3 |
  implE : {A,B} ⊢ A ⇒ B → ⊢ A → ⊢ B | # implE 3 4 |

  orIl : {A,B} ⊢ A → ⊢ A ∨ B | # orIl 3 |
  orIr : {A,B} ⊢ B → ⊢ A ∨ B | # orIr 3 |
  orE : {A,B,C} ⊢ A ∨ B → ( ⊢ A → ⊢ C ) → ( ⊢ B → ⊢ C ) → ⊢ C | # orE 4 5 6 |

  notI : {A} ( ⊢ A → ⊢ ⊥ ) → ⊢ ¬A | # notI 2 |
  notE : {A} ⊢ ¬¬A → ⊢ A | # notE 2 |
```

Example 3.10 (Propositional Logic (Natural Deduction)).

```
theory Proofs : ?PropLogNatDed =
  conjComm : {A,B} ⊢ A ∧ B ⇒ B ∧ A |
  = [A,B] implI ([ab] andI (andEr ab) (andEl ab)) |
```
Example 3.11 (First-Order Logic (Syntax)).

```
theory FOLSyntax : ur:LF =
  include ?PropLogSyntax |

ind : type | # i | /T jiota |

forall : (i → bool) → bool | # ∀ 1 prec 55 |
exists : (i → bool) → bool | # ∃ 1 prec 60 |
   = [P] → ∀ [x] → (P x) | /T jexists

// existsUnique : ??? |- ??? | # ∃! 1 prec 65 |
```

```
theory FOLEQSyntax : ur:LF =
  include ?FOLSyntax |
  equality : i → i → bool | # 1 ÷ 2 prec 65 |
```
Example 3.12 (First-Order Logic (Natural Deduction)).

theory FOLNatDed : ur:?LF =
  include ?FOLSyntax |
  include ?PropLogNatDed |

forallI : {P} (\{y : t\} ⊢ P y) ⊢ ∀ [x] P x | # forallI 2 |
forallE : {P,B} ⊢ (∀ [x] P x) ⊢ P B | # forallE 3 |
/T Everytime you write ∀ P$, somewhere a unicorn cries |

existsI : {P,c} ⊢ (P c) ⊢ ∃ [x] P x | # existsI 3 |
existsE : {P,B} ⊢ (∃ [x] P x) ⊢ (\{c\} ⊢ P c ⊢ B) ⊢ B | # existsE 3 4 |
4 MMT Software Ecosystem
MMT: Universal representation language for formal mathematical/logical content

Implementation: MMT API with generic
- module system for math libraries, logics, foundations
- parsing + type reconstruction + simplification
- IDEs
- change management

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Close relatives:
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MMT API JEdit Integration (IDE)
Kohlhase: MMT as a Logic/Language/*-Workbench

MMT API IntelliJ (IDE)
MMT API Browser Integration

The MMT Web Server

Style: html5

cds.ommoc.org / courses / 2013 / ACS1 / exercise_10.mmt?Problem3

theory Problem3 meta LF

include: http://cds.ommoc.org/examples?FOLEQNatDed

circ : term → term → term

e : term

R : ⊢ ∀x e ∈ x

C : ⊢ ∀x ∀y e + y = y + x

L : ⊢ ∀x e = x

reconstructed types

implicit arguments

redundant brackets

show

hide

Enter an object over theory: http://cds.ommoc.org/courses/2013

[x] x ∈ e

alyze simplify

[x] x + e

{x : term} term
MathHub: A Portal and Archive of Flexiformal Maths

- **Idea**: learn from the open source community, offer a code repository with management support that acts as a hub for publication/development projects.

- **MathHub**: a collaborative development/hosting/publishing system of open-source, formal/informal math. (See [http://mathhub.info](http://mathhub.info))
MathHub: A Portal and Archive of Flexiformal Maths

▶ Idea: learn from the open source community, offer a code repository with management support that acts as a hub for publication/development projects.

▶ MathHub: a collaborative development/hosting/publishing system of open-source, formal/informal math. (See http://mathhub.info)

▶ MathHub Architecture: Three core components (meet requirements above)
  ▶ Representation: OMDoc/MMT mechanized by the MMT system.
  ▶ Repositories: GitLab (git-based public/private repositories)
  ▶ Front-End: React.JS (all content served by MMT)
Definition 4.1. TGView is a flexible facility for viewing and interacting with (theory) graphs in MathHub.

- TGView gives access to MathHub libraries
- MMT API generates JSON graph representation
- TGView draws graph to Browser canvas (via the vis.js library)

TGView3D is a VR version for the Oculus Rift.

Example 4.2 (CAS Interfaces, MitM Ontology, and Alignments).
5 MMT+GF as a Natural Language Semantics Workbench
Meaning of Natural Language; e.g. Machine Translation

- **Idee:** Machine Translation is very simple! (we have good lexica)

- **Example 5.1.** Peter liebt Maria. \(\sim\) Peter loves Mary.

- **Example 5.2.** Wirf der Kuh das Heu über den Zaun. \(\nRightarrow\) Throw the cow the hay over the fence. (differing grammar; Google Translate)

- **Example 5.3.** \(\nRightarrow\) Grammar is not the only problem
  - Der Geist ist willig, aber das Fleisch ist schwach!
  - Der Schnaps ist gut, aber der Braten ist verkocht!

- We have to understand the meaning!
Observation: Humans use words (sentences, texts) in natural languages to represent and communicate information.

But: what really counts is not the words themselves, but the meaning information they carry.
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**Example 5.4.**

\( \text{Zeitung} \sim \)

**for questions/answers, it would be very useful to find out what words (sentences/texts) mean.**
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But: what really counts is not the words themselves, but the meaning information they carry.

Example 5.4.

Zeitung

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Interpretation of natural language utterances: three problems

- schema
- abstraction
- ambiguity
- composition
Example 5.5 (Abstraction).

*car* and *automobile* have the same meaning
Example 5.5 (Abstraction).

Car and automobile have the same meaning.

Example 5.6 (Ambiguity).

A bank can be a financial institution or a geographical feature.
Language and Information (Examples)

Example 5.5 (Abstraction).

*car* and *automobile* have the same meaning

Example 5.6 (Ambiguity).

*a bank* can be a financial institution or a geographical feature

Example 5.7 (Composition).

*Every student sleeps* $\sim \forall x.\text{student}(x) \Rightarrow \text{sleep}(x)$
Observation: Not all information conveyed is linguistically realized in an utterance.

Example 5.8. *The lecture begins at 11:00 am.* What lecture? Today?

Definition 5.9. We call a piece $i$ of information linguistically realized in an utterance $U$, iff, we can trace $i$ to a fragment of $U$.

Possible Mechanism: Inference

Utterance $\rightarrow$ Meaning $\rightarrow$ relevant information of utterance

Grammar $\rightarrow$ Lexicon $\rightarrow$ World knowledge $\rightarrow$ Inference
Example 5.10. *It starts at eleven.* What starts?
Before we can resolve the time, we need to resolve the anaphor *it*.

Possible Mechanism: More Inference!
# What is the State of the Art In NLU?

- **Two avenues of attack for the problem**: knowledge-based and statistical techniques *(they are complementary)*

<table>
<thead>
<tr>
<th>Deep</th>
<th>Knowledge-based</th>
<th>Not there yet cooperation?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shallow</td>
<td>no-one wants this</td>
<td>Statistical Methods applications</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Analysis ↑</th>
<th>narrow</th>
<th>wide</th>
</tr>
</thead>
<tbody>
<tr>
<td>vs.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coverage →</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- We will cover foundational methods of deep processing in the course and a mixture of deep and shallow ones in the lab.
Environmental Niches for both Approaches to NLU

- There are two kinds of applications/tasks in NLU
  - consumer-grade applications have tasks that must be fully generic, and wide coverage (e.g. machine translation \(\sim\) Google Translate)
  - producer-grade applications must be high-precision, but domain-adapted (multilingual documentation, voice-control, ambulance translation)

<table>
<thead>
<tr>
<th>Precision</th>
<th>Producer Tasks</th>
<th>Consumer Tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>100%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- A producer domain I am interested in: Mathematical/Technical documents
Natural Language Semantics?

$\mathcal{M} = \langle D, I \rangle$ induces $\models \subseteq \mathcal{FL} \times \mathcal{FL}$

$\models \equiv \vdash_C$?

$\models_\mathcal{NL} \equiv \vdash_C$?

$\mathcal{M} = \langle D, I \rangle$ induces $\mathcal{N}\mathcal{L} \subseteq \mathcal{N}\mathcal{L} \times \mathcal{N}\mathcal{L}$

$\mathcal{N}\mathcal{L}$

Compl Ling

Analysis

$\mathcal{I}_\varphi$

$\mathcal{L} = \text{wff} (\Sigma)$

formulae

$\vdash_C \subseteq \mathcal{FL} \times \mathcal{FL}$

$\models \subseteq \mathcal{FL} \times \mathcal{FL}$

$\mathcal{N}\mathcal{L}$

Definition 5.11. Fragment 1 knows the following eight syntactical categories:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>sentence</td>
</tr>
<tr>
<td>$N$</td>
<td>noun</td>
</tr>
<tr>
<td>$V^i$</td>
<td>intransitive verb</td>
</tr>
<tr>
<td>conj</td>
<td>connective</td>
</tr>
<tr>
<td>$NP$</td>
<td>noun phrase</td>
</tr>
<tr>
<td>$N_{pr}$</td>
<td>proper name</td>
</tr>
<tr>
<td>$V^t$</td>
<td>transitive verb</td>
</tr>
<tr>
<td>Adj</td>
<td>adjective</td>
</tr>
</tbody>
</table>

Definition 5.12. We have the following grammar rules in fragment 1:

<table>
<thead>
<tr>
<th>Rule</th>
<th>Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1.</td>
<td>$S \rightarrow NP \ V^i$</td>
</tr>
<tr>
<td>S2.</td>
<td>$S \rightarrow NP \ V^t \ NP$</td>
</tr>
<tr>
<td>N1.</td>
<td>$NP \rightarrow N_{pr}$</td>
</tr>
<tr>
<td>N2.</td>
<td>$NP \rightarrow \text{the}N$</td>
</tr>
<tr>
<td>S3.</td>
<td>$S \rightarrow \text{It is not the case that } S$</td>
</tr>
<tr>
<td>S4.</td>
<td>$S \rightarrow S \ conj \ S$</td>
</tr>
<tr>
<td>S5.</td>
<td>$S \rightarrow NP \ is \ NP$</td>
</tr>
<tr>
<td>S6.</td>
<td>$S \rightarrow NP \ is \ Adj.$</td>
</tr>
</tbody>
</table>
Syntax Example: *Jo poisoned the dog and Ethel laughed*

- **Observation 5.13.** *Jo poisoned the dog and Ethel laughed* is a sentence of fragment 1

- We can construct a syntax tree for it!
Example 5.14 (Propositional Logic (Syntax)).

theory PropLogSyntax : ur:?LF =
  prop : type | # bool |

and : bool → bool → bool | # 1 ∧ 2 prec 45 | /T jwedge |
not : bool → bool | # ¬ 1 prec 50 | /T jneg |
or : bool → bool → bool | # 1 ∨ 2 prec 40 |
  = [a,b] ∨ (¬ a ∧ ¬ b) | /T jvee |
implies : bool → bool → bool | # 1 → 2 prec 35 |
  = [a,b] ↝ a v b | /T jR |
iff : bool → bool → bool | # 1 ⇔ 2 prec 40 |
  = [a,b] (a → b) ∧ (b → a) |
true : bool | # ⊤ | /T jtop |
false : bool | = ⊥ | # ⊥ | /T jbot |
A “lexicon theory”

\[
\begin{align*}
\text{theory frag1Lex : ?plnqd} & = \\
\text{meta ?gfmeta?correspondsTo `frag1Lex.pgf} & \\
\text{Ethel_NP : } & t \\
\text{book_N : } & \text{pred1} \\
\text{sing_V : } & \text{pred1} \\
\text{read_V2 : } & \text{pred2} \\
\text{happy_A : } & \text{pred1} \\
\end{align*}
\]

declares one logical constant for each from abstract GF grammar (automation?)

Extend by axioms that encode background knowledge about the domain

Example 5.15 (What makes you sing).

\[
\begin{align*}
\text{happy_sing : } & \vdash \forall [x] \text{ happy } x \Rightarrow \text{ sing } x \\
\text{read_happy : } & \vdash \forall [x] \ (\exists [y] \text{ book } y \land \text{ read } x y) \Rightarrow \text{ happy } x
\end{align*}
\]
Example 5.16 (A Hello World Grammar).

```plaintext
abstract zero = {
flags startcat=O;
cat
S ; NP ; V2 ;
fun
spo : V2 -> NP -> NP -> S ;
John, Mary : NP ;
Love : V2 ;
}

concrete zeroEng of zero = {
lincat
S, NP, V2 = Str ;
lin
spo vp s o = s ++ vp ++ o;
John = "John" ;
Mary = "Mary" ;
Love = "loves" ;
}
```

- parse a sentence in gf: parse "John loves Mary" \(\leadsto\) Love John Mary
Hello World Example for GF (Syntactic)

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- Make a French grammar with John="Jean"; Mary="Marie"; Love="aime";
- Parse a sentence in gf: parse "John loves Mary" ~ Love John Mary
Hello World Example for GF (Syntactic)

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    Love = "loves" ;
}
```

- Make a French grammar with John="Jean"; Mary="Marie"; Love="aime"
- parse a sentence in gf: parse "John loves Mary" \(\leadsto\) Love John Mary
- linearize in gf: linearize Love John Mary \(\leadsto\) John loves Mary
- translate in in gf: parse \(-lang=Eng\) "John Loves Mary" | linearize \(-lang=Fre\)
- generate random sentences to test:
  generate_random \(-number=10\) | linearize \(-lang=Fre\) \(\leadsto\) Jean aime Marie
Embedding GF into MMT

- **Observation:** GF provides Java bindings and MMT is programed in Scala, which compiles into the Java virtual machine.

- **Idea:** Use GF as a sophisticated NL-parser/generator for MMT
  - MMT with a natural language front-end.
  - GF with a multi-logic back-end

- **Definition 5.17.** The GF/MMT integration mapping interprets GF abstract syntax trees as MMT terms.

- **Observation:** This fits very well with our interpretation process in LBS

- **Implementation:** transform GF (Java) data structures to MMT (Scala) ones
Correspondence between GF Grammars and MMT Theories

- **Idea:** We can make the GF/MMT integration mapping essentially the identity.

- **Prerequisite:** MMT theory isomorphic to GF grammar (declarations aligned)

- **Mechanism:** use the MMT metadata mechanism
  - symbol `correspondsTo` in metadata theory `gfmeta` specifies relation
  - import `gfmeta` into domain theories
  - meta keyword for “metadata relation whose subject is this theory”.
  - object is MMT string literal ‘`grammar.pgf`.

```plaintext
3  theory gfmeta : ur:?LF = correspondsTo  \\
4  theory plnqd : ur:?LF =  \\
6  include ?gfmeta  \\
7  meta ?gfmeta?correspondsTo `grammar.pgf`
```

- **Observation:** GF grammars and MMT theories best when organized modularly.

- **Best Practice:** align “grammar modules” and “little theories” modularly.
6 OMDoc/MMT in Argumentation Theory
6.1 Introduction: Argumentation Theory
[adapted from Sarah Gaggl]
Argumentation is Ubiquitous

- **Observation**: We exchange arguments in politics, in court, when making decisions, and in science
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- **Observation**: We exchange arguments in politics, in court, when making decisions, and in science

- **Questions**: But what is argumentation? Can we model/decide arguments?
Argumentation is Ubiquitous

- **Observation**: We exchange arguments in politics, in court, when making decisions, and in science
- **Questions**: But what is argumentation? Can we model/decide arguments?
- **Example 6.1**: Is this Argumentation?
Background: SPP 1999 RATIO & Project ALMANAC

- DFG Schwerpunktprogramm (SPP) 1999 (established 2017)
- RATIO: Robust Argumentation Machines (2018-20; 2021-23)
- Going from mere facts to coherent argumentative structures as information units for decision-making
- Areas involved: semantic web, computational linguistics, information retrieval, Logic, human/computer interaction.

ALMANAC: Argumentation Logics Manager & Argument Context Graph,
- WA1: Atlas of Argumentation Logics (representing/organizing logics in LF)
- WP2: Context Graphs for Argumentation (Theory Graphs for Multi-Agent-Logic)
- WP3: Archiving & Managing Argumentation Logics (MathHub.info)

Kohlhase: MMT as a Logic/Language/*-Workbench
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- ca. 12 projects, (see http://spp-ratio.de)
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Definition 6.2 (Plato’s Dialectic).
The dialectical method is discourse between two or more people holding different points of view about a subject, who wish to establish the truth of the matter guided by reasoned arguments. (The Republic (Plato), 348b)

Definition 6.3 (Leibniz’ Dream).
The only way to rectify our reasonings is to make them as tangible as those of the Mathematicians, so that we can find our error at a glance, and when there are disputes among persons, we can simply say: Let us calculate [Calculemus!], without further ado, to see who is right. (Leibniz, Gottfried Wilhelm, The Art of Discovery 1685, Wiener 51)
Abstract Argumentation Systems

- **Abstract Argumentation [Dung, 1995]:**
  - In abstract argumentation frameworks (AAF) statements (called arguments) are formulated together with a relation (attack) between them.
  - Abstraction from the internal structure of the arguments.
  - The conflicts between the arguments are resolved on the semantical level.

- **Example 6.4.**

Situation

![Situation Image]

Result

![Result Image]
Legal Reasoning
Decision Support
The Problem with Abstract Argumentation Systems

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- **Example 6.5.**

  ![Situation](situation.png) ![Result](result.png)
Abstract Argumentation [Dung, 1995]:

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**Example 6.6.**

![Diagram of the example](image-url)
Robust Representation of Individual Inference

- **Idea**: To represent arguments, we need to represent everyday reasoning.
- *There is a logic for that!* (actually many many of them)
Robust Representation of Individual Inference

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- **There is a logic for that!** (actually many many of them)
- Robust Representation of Individual Inference (usually “philosophical logics”)
  - (multi-)modal logics extend classical logic by notions of possibility and necessity.
  - Preference logic allows for stating sentences of the form “A is better/worse than B”. [Han02]
  - Relevance logic restricts the classical (i.e. material) implication to protect from implications between seemingly disconnected premises and conclusions, [DR02].
  - Other paraconsistent logics, which try to deal with inconsistency in a non-fatal manner by systematically avoiding *ex falso quodlibet*.
  - Temporal logics allow for reasoning about time (e.g. “X is true at time $t_0$”), [Bur84],
  - Probabilistic logics about probabilities. [Nil86].
  - Dynamic Logics to model all kinds of anaphora
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- **Proof Theory**: Most logics have a natural-deduction-style calculus, some even machine-oriented calculi.

- **Model Theory**: mostly modal \( \models \) possible worlds semantics
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- **Proof Theory**: Most logics have a natural-deduction-style calculus, some even machine-oriented calculi.

- **Model Theory**: mostly modal $\sim$ possible worlds semantics

- **Interoperability Problem**: Most logics are “formally unrelated”, incomparable (evaluation?, duplicated work)
6.2 Work Area 2: Context Graphs for Argumentation
Deep Modeling of Argumentation in STEM Settings

- **Observation**: Much of the wealth and prospects of central European Countries are based on STEM knowledge. (laid down in technical documents)
- STEM documents often have a non-trivial argumentation structure

Example 6.7. Short excerpt of Coffey’s and Sondow’s rebuttal [CS12] of Kowalenko’s paper [Kow10].

The irrationality of Euler’s constant $\gamma$ has long been conjectured. In 2010, Kowalenko claimed that simple arguments suffice to settle this matter [4].

We describe the flaws in his very limited approach. Kowalenko derives the following formula for Euler’s constant in equation (65) of [4, p. 428]:

$$C - \frac{\pi^2}{6}$$

Here he claims that the sum of a series of positive rational numbers cannot be equal to $C - \frac{\pi^2}{6}$. But, for example, decimal expansion does give such a series:

- **Observation**: Often the aim of STEM argumentation is uncovering the truth (and reputation/grant money gain)
- **Idea**: RATIO on technical/scientific documents (needs deep modeling)
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► Observation: Often the aim of STEM argumentation is **uncovering the truth** (and reputation/grant money gain)

► Idea: RATIO on technical/scientific documents (needs deep modeling)
Deep Modeling of Argumentation in STEM Settings

- **Observation**: often the ultimate source of differing opinions in STEM lies in differing assumptions.

- **Example 6.8 (Example)**: various models in physics that make differing predictions, e.g. heliocentric vs. geocentric universe.

- **Scientific Method**: Explore the inferential closure of the model assumptions, contrast to others/experiments, argue for your model.

- **Idea**: Meta-model differing model assumptions as OMDoc/MMT theory graph
  - recast the *support*, *refutation* or *undercut* relations via theory morphisms + $\epsilon$.
  - theory morphisms incorporate inferential closure and *renaming/framing*.
  - concept-minimal graphs explicitly manage common ground.
  - Extend theory graph algorithms for that.
Modular Representation of Math (MMT Example)

\[
\begin{align*}
\text{IntArith} & : \mathbb{Z} := \mathbb{P}/\mathbb{N} \cup \mathbb{N}/\mathbb{N} \\
& \quad = 0
\end{align*}
\]

\[
\begin{align*}
\text{NatTimes} & : n \cdot 1 = n, \\
& \quad n \cdot s(m) = n \cdot m + n
\end{align*}
\]

\[
\begin{align*}
\text{NatPlus} & : n + 0 = n, \\
& \quad n + s(m) = s(n + m)
\end{align*}
\]

\[
\begin{align*}
\text{NatNums} & : \mathbb{N}, s, 0 \\
& \quad P_1, \ldots P_5
\end{align*}
\]

\[
\varphi = \{ G \mapsto \mathbb{N}, o \mapsto \cdot, e \mapsto 1 \}
\]

\[
\psi = \{ G \mapsto \mathbb{N}, o \mapsto +, e \mapsto 0 \}
\]

\[
\varphi' = \{ i \mapsto -, g \mapsto f \}
\]

\[
\psi' = \{ x \mapsto e \}
\]

\[
\psi = \{ x \mapsto e, y \mapsto c \}
\]

\[
\begin{align*}
\text{Ring} & : (x \cdot o \cdot y) \cdot o \cdot z = (x \cdot o \cdot y) \cdot o \cdot (x \cdot o \cdot z) \\
\text{Monoid} & : e \circ x = x \\
\text{Semigroup} & : \text{assoc}:(x \circ y) \circ z = x \circ (y \circ z)
\end{align*}
\]

\[
\begin{align*}
\text{Group} & : i = \lambda x. \tau y. x \circ y = e \\
\text{CGroup} & : \text{comm} : x \circ y = y \circ x
\end{align*}
\]

\[
\begin{align*}
\text{NonGrpMon} & : \exists x : G. \forall y : G. x \circ y \neq e \\
\text{Group} & : \forall x : G. \exists y : G. x \circ y = e
\end{align*}
\]
Framing in Arguments

Definition: In a nutshell, framing means that a concept mapping between argumentation/knowledge contexts (a frame) is established and the facts and assumptions underlying the argument are mapped along the frame.

Observation: This happens often in counter-arguments by framing the original argument in terms of an obviously wrong argument.

Example 6.9 (Roe vs. Wade). from www.truthmapping.com/map/647/

- The 1973 Roe vs. Wade decision denied fetus’ rights on the basis of personhood.
- The 1857 Dred Scott decision denied Black Americans rights on the basis of personhood.
- Personhood for Black Americans has been denied purely on the basis of cultural consensus.
- Therefore the denial of personhood for fetuses could also be purely on the basis of cultural consensus.

Model in a theory graph using a frames as morphisms approach

\[ \varphi: \{\text{DredScott1857} = \text{RoevsWade1973} \}
\text{black} = \text{fetus} \}
\]

- Arg1 {RoevsWade1973 : Court Decision
  P1 : RoevsWade1973 ⇒ ¬ Person(fetus)
  Conclusion : ¬ Rights(fetus)}

- Arg2 {DredScott1857 : Court Decision
  P1 : DredScott1857 ⇒ ¬ Person(black)
  Conclusion : ¬ Rights(black)}

CG {\forall x.¬ Person(x) ⇒ ¬ Rights(x)}
Work Area 2: Work Plan

- WP2.1: Annotated Corpus of Technical Documents
  1. Subcorpus Identification
  2. Argumentation/Context Annotation
  3. Distribution

- WP2.2: Context Graph via Argumentation Relations

- WP2.3: Extending the MMT system with Context Graph Relations

- WP2.4: Framing in Arguments
  1. Modelling (work through lots of examples)
  2. Automation (use the OMDoc/MMT view finder to discover possible frames)
Visual Conclusion (please ask questions)

▶ Summary: Understanding/Supporting Logic-Based Deep Modeling of Arg.

▶ Contribution: develop and manage the targets of semantics extraction!
7 Application: Serious Games
Example 7.1 (Problem 0.8.15).

How can you measure the height of a tree you cannot climb, when you only have a protactor and a tape measure at hand.
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How can you measure the height of a tree you cannot climb, when you only have a protractor and a tape measure at hand.

Framing: view the problem as one that is already understood (using theory morphisms)

- squiggly (framing) morphisms guaranteed by metatheory of theories!
Example Learning Object Graph

Game World

User Knowledge

MMT

New Knowledge

Game Problem

Fact Discovery

Explored World

Interaction

Game Solution

Solution Pushout

Situation Theory

Solution Theory

Problem Theory

Planar Geometry

Forestry

vertical (tree) horizontal (ground) ...

Planar Geometry

point : typeline : point → point → line |ab| : line → R ⊥ : line → line → bool ...

FrameIT Method: Problem

- Problem Representation in the game world (what the student should see)

- Student can interact with the environment via gadgets so solve problems
- “Scrolls” of mathematical knowledge give hints.

What is the height of this tree?
Combining Problem/Solution Pairs

- We can use the same mechanism for combining P/S pairs
- create more complex P/S pairs (e.g. for trees on slopes)
## Overview: KWARC Research and Projects

### Applications:
- eMath 3.0
- Active Documents
- Semantic Spreadsheets
- Semantic CAD/CAM
- Change Management
- Global Digital Math Library
- Math Search Systems
- SMGloM: Semantic Multilingual Math Glossary
- Serious Games

### Foundations of Math:
- MathML, *OpenMath*
- Advanced Type Theories
- MMT: Meta Meta Theory
- Logic Morphisms/Atlas
- Theorem Prover/CAS Interoperability
- Mathematical Models/Simulation

### KM & Interaction:
- Semantic Interpretation (aka. Framing)
- Math-literate interaction
- MathHub: math archives & active docs
- Semantic Alliance: embedded semantic services

### Semantization:
- \( \LaTeX \rightarrow XML \)
- \( \LaTeX \rightarrow \) invasive editors
- Context-Aware IDEs
- Mathematical Corpora
- Linguistics of Math
- ML for Math Semantics Extraction

### Foundations:
- Computational Logic
- Web Technologies
- *OMDoc/MMT*


Peder Olesen Larsen and Markus von Ins. “The rate of growth in scientific publication and the decline in coverage provided by Science Citation Index”. In: *Scientometrics* 84.3 (2010), pp. 575–603. DOI: 10.1007/s11192-010-0202-z.