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Stochastics II

11. Tutorial

Exercise 1

The classical deterministic exponential function $f(t) = \exp(t)$ has the remarkable property $f'(t) = f(t)$ or equivalently $f(t) - f(s) = \int_s^t f(x)dx$. Now we are looking for a stochastic process X with

$$X_t - X_s = \int_s^t X_x dB_x, \quad s \leq t,$$

where (B_t) is the standard Brownian motion.

(a) Show that $f(B_t) = \exp(B_t)$ does not satisfy the above equation.

(b) Find a suitable function $f(t, B_t)$ with

$$f(t, B_t) - f(s, B_s) = \int_s^t f(x, B_x) dB_x.$$

Hint: Use Ito's lemma.

Exercise 2

Let B_t be a standard Brownian motion and let S_t be a stochastic process that satisfies the SDE

$$dS_t = \mu S_t dt + \sigma S_t dB_t, \quad S_0 = s_0 \in \mathbb{R}_+.$$

(i) Assume S_t models a stock price and interpret the parameters μ and σ.

(ii) Solve the SDE given above.

Hint: Use the following general result:
Assume X_t is an Itô process, i.e. $dX_t = \mu(t, X_t)dt + \sigma(t, X_t)dB_t$ and $Y_t = f(t, X_t)$, where f is twice continuously differentiable on $[0, \infty] \times \mathbb{R}$, then Y_t is also an Itô process given by

$$dY_t = \left(f_x(t, X_t)\mu(t, X_t) + f_t(t, X_t) + \frac{1}{2} f_{xx}(t, X_t)\sigma^2(t, X_t) \right) dt + f_x(t, X_t)\sigma(t, X_t)dB_t.$$

Apply the result for a suitable function $f(t, S_t)$ with $f_t(t, X_t) = 0$.

(iii) Calculate, given the parameters $\mu = 0.25$, $\sigma = 0.2$ on an annual basis, the probability that the stock price will exceed 45 in four months' time given that its current price is 38.

Hint: For a fixed t, $B_t \sim N(0,t)$.

Exercise 3

Assume B_t is a standard Brownian motion. Use the Itô isometry in order to compute the variances of the following stochastic integrals

(a) $\int_0^t |B_s|^{\frac{1}{2}} dB_s$,

Hint: Note $E\left(\int_0^t |B_s|^{\frac{1}{2}} dB_s\right) = 0$.

(b) $\int_0^t (B_s + s)^2 dB_s$.

Hand in **We 26.01.11 up to 15.00** in postbox 20 at F4.