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Stochastic II

8. Tutorial

Exercise 1
Let $(B_t)_{t\geq 0}$ be a standard Brownian motion. Apply Itô's Lemma in order to show that the following stochastic processes are martingales

(i) $X_t = e^{\frac{1}{2}t}\cos B_t$

(ii) $X_t = e^{\frac{1}{2}t}\sin B_t$

Exercise 2
Let $(B_t)_{t\in[0,T]}$ be a standard brownian motion. Show if g is a continuous differentiable function the process

$$M_t = g(t)B_t - \int_0^t g'(s)B_s ds$$

is a martingale.

Hint: Define a suitable function f(t,x) and apply Ito's formula. Note that $\int g(t)dB_t$ *is a martingale.*

Exercise 3
Let $(B_t)_{t\in[0,T]}$ be a standard brownian motion and let x_0, $a \in \mathbb{R}$. Show that the process $(X_t)_{t\in[0,T]}$ defined by

$$X_t = \exp(-at)x_0 + \int_0^t \exp(-a(t-s))dB_s$$

is a solution of the stochastic differential equation

$$dX_t = -aX_t dt + dB_t,\ X_0 = 0.$$

Hand in **We 15.12.10 up to 16.00** in postbox 20 at F4.