University of Konstanz
Department of Mathematics and Statistics
Prof. Dr. M. Kohlmann
Yevgen Shumeyko 12.01.2011

Stochastics II

10. Tutorial

Exercise 1

(i) Solve the following SDE for the spot rate of interest

$$dr_t = a(b - r_t)dt + \sigma dB_t$$

Hint: First, set $\sigma = 0$ and solve the deterministic differential equation $d(r_t - b) = a(b - r_t)dt$. Generalize the solution such that it solves the full equation.

(ii) What is the probability distribution of the solution r_t? What is the limit of this distribution as $t \to \infty$?

(iii) Given the parameters $a = 150\%$ p.a., $b = 3\%$ and $\sigma = 6\%$ on a annual basis, what is the probability that the rate of interest will be less than 4% per annum in six month time given that its current value is 2.5%?

Exercise 2
Find the SDEs satisfied by the processes:

(a) $X_t = e^t \sin B_t$,

(b) $X_t = \sin tB_t$,

(c) $X_t = a \cos B_t$, $Y_t = b \sin B_t$, where $ab \neq 0$.

Exercise 3
Let B_t be a standard Brownian motion. Find a solution process $X_t := f(t, B_t)$ for each of the following SDEs

(a) $dX_t = B_t\, dt + t\, dB_t$,

Hint: Note $\frac{\partial^2 f(t,B_t)}{(\partial x)^2} = f_{xx}(t, B_t) = 0$.

(b) $(1+t)\ dX_t = -X_t\ dt + dB_t,$

Hint: Note $\frac{\partial^2 f(t,B_t)}{(\partial x)^2} = f_{xx}(t, B_t) = 0.$

(c) $dX_t = -\frac{1}{2} X_t dt + \sqrt{1 - X_t^2} dB_t.$

Hint: Note $X_t := f(B_t),\ i.e.\ \frac{\partial f(t,B_t)}{\partial t} = f_t(t, B_t) = 0.$

Hand in **We 19.01.11 up to 15.00** in postbox 20 at F4.