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Tutorials for 'Real Closed Fields and Integer Parts'

Bonus Sheet

General Note: Solutions of this sheet will not be marked. If you have any questions, you may write us an e-mail or come to our offices.

Exercise A (Overspill for IOpen)

Let $\mathcal{M} \models \mathrm{IOpen}$ be non-standard, let $\varphi(x, \underline{y})$ be a quantifier-free $\mathcal{L}_{\mathrm{PA}}$-formula and let $\underline{a} \in M$. Suppose that $\mathcal{M} \models \varphi(n, \underline{a})$ for any standard element $n \in M$. Show that there exists a non-standard element $m \in M$ such that $\mathcal{M} \models \varphi(m, \underline{a})$.

Exercise B (Real Closed Fields and o-Minimality)

Show that an ordered field $(K, <)$ is o-minimal if and only if it is real closed.

Exercise C (Integer Parts)

Find a real closed field $(K, <)$ and a discretely ordered subring Z of K such that Z is cofinal in K but Z is not an integer part of K.

Definition D. Let $(A, <)$ be a linear ordering. A subset $C \subseteq A$ is called **cofinal** in A if for any $a \in A$ there exists $c \in C$ such that $a \leq c$.