Prof. Dr. Salma Kuhlmann
Laura Wirth

Universität Konstanz
Fachbereich Mathematik und Statistik
Sommersemester 2026

Valued Fields

Exercise Sheet 3
Valuation Independence

Let Q be a field. If not further specified, any vector space we consider is a Q-vector space.

Exercise 3.1. (2+2 points)

Let (V_1, v_1) and (V_2, v_2) be valued vector spaces such that $S(V_1) = S(V_2)$. Let $h\colon V_1 \to V_2$ be a valuation preserving isomorphism of vector spaces, i.e. for any $a \in V_1$, we have $v_2(h(a)) = v_1(a)$. Let $\mathcal{B} \subseteq V_1 \setminus \{0\}$.

(a) Show that $\mathcal{B}$ is Q-valuation independent if and only if $h(\mathcal{B})$ is Q-valuation independent.

(b) Show that $\mathcal{B}$ is a Q-valuation basis for (V_1, v_1) if and only if $h(\mathcal{B})$ is a Q-valuation basis for (V_2, v_2).

Exercise 3.2. (4 points)

Let (V_1, v_1) and (V_2, v_2) be valued vector spaces such that $S(V_1) = S(V_2)$. Let $\mathcal{B}_1 \subseteq V_1 \setminus \{0\}$ be a Q-valuation basis for (V_1, v_1) and let $\mathcal{B}_2 \subseteq V_2 \setminus \{0\}$ be a Q-valuation basis for (V_2, v_2). Suppose that there exists a valuation preserving bijection

$$\widetilde{h}\colon \mathcal{B}_1 \to \mathcal{B}_2.$$

Let $h\colon V_1 \to V_2$ be the isomorphism obtained by linearly extending $\widetilde{h}$.
Show that h is valuation preserving.

Exercise 3.3. (1+1+1+1 points)

Consider the $\mathbb{Q}$-vector space $(V, v) = (\mathbf{H}_{n\in\mathbb{N}} B_n, v_{\min})$.

(a) Let $B_n = \mathbb{Q}$ for any $n \in \mathbb{N}$.

(i) Describe a maximal $\mathbb{Q}$-valuation independent set $\mathcal{B} \subseteq V$ such that $\operatorname{support}(b)$ is a singleton for any $b \in \mathcal{B}$.

(ii) Describe a maximal $\mathbb{Q}$-valuation independent set $\mathcal{B} \subseteq V$ such that $\operatorname{support}(b)$ is infinite for any $b \in \mathcal{B}$.

(b) Let $B_n = \mathbb{R}$ for any $n \in \mathbb{N}$.

(i) Describe a maximal $\mathbb{Q}$-valuation independent set $\mathcal{B} \subseteq V$ such that $\mathrm{support}(b)$ is a singleton for any $b \in \mathcal{B}$.

(ii) Describe a maximal $\mathbb{Q}$-valuation independent set $\mathcal{B} \subseteq V$ such that $\mathrm{support}(b)$ is infinite for any $b \in \mathcal{B}$.

Submission:
Please hand in your solutions by **Tuesday, 5 May 2026, 10:00h** (postbox 17).