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Fachbereich Mathematik und Statistik
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Valued Fields

Exercise Sheet 11
Convex Valuations

Exercise 11.1. (5 points)

Let K be a field with valuations w_1 and w_2.

(a) Show that the following are equivalent:

(i) w_2 is coarser than w_1.

(ii) $I_{w_2} \subseteq I_{w_1}$.

(iii) For any $a, b \in K$, if $w_1(a) \leq w_1(b)$, then $w_2(a) \leq w_2(b)$.

(b) Suppose that w_2 is coarser than w_1. Let

$$\varphi\colon\ K_{w_2} \to Kw_2, a \mapsto aw_2$$

be the residue map of w_2, where K_{w_2} denotes the valuation ring and Kw_2 the residue field of (K, w_2). Show that $\varphi(K_{w_1})$ is a valuation ring of the residue field Kw_2.

Exercise 11.2. (4 points)

(a) Let $\mathbb{K} = \mathbb{R}((\mathbb{Q} \times \mathbb{R}))$, where $\mathbb{Q} \times \mathbb{R}$ is ordered lexicographically. Let

$$C = \{(0, z) \mid z \in \mathbb{R}\}.$$

(i) Compute the convex valuation w on $\mathbb{K}$ associated to C.

(ii) Find the value group and the residue field of $(\mathbb{K}, w)$.

(iii) Compute the rank of $\mathbb{K}$.

(b) Let $K = \mathbb{R}(t)$. Show that for any ordering on K the rank of K is a singleton with $\mathcal{R} = \{K\}$.

Submission:
Please hand in your solutions by **Tuesday, 07 July 2026, 10:00h** (postbox 17).