
Valued Fields
Exercise Sheet 12
Valuations on Power Series Fields

Exercise 12.1. (16 points)

Let $G \neq \{0\}$ be a divisible ordered abelian group and let $H_2 \subseteq G$ be a non-trivial proper convex subgroup.

- (a) Show that for some divisible ordered abelian group H_1 , there exists an order-preserving isomorphism

$$\varphi: G \rightarrow H,$$

where $H = H_1 \times H_2$ is ordered lexicographically. Moreover, show that $\varphi(H_2)$ is a convex subgroup of H .

(Hint: Consider G as a $\mathbb{Q}$ -vector space.)

- (b) Let $K = \mathbb{R}((H_2))$. Find an order-preserving isomorphism of fields

$$\psi: \mathbb{R}((H)) \rightarrow K((H_1)).$$

- (c) Let w be the valuation on $\mathbb{R}((H))$ associated to the convex subgroup $\varphi(H_2)$ of H .

- (i) Find an order-preserving embedding of groups from the value group $w(\mathbb{R}((H))^\times)$ into $(\mathbb{R}((H))^{>0}, \cdot, 1, <)$.
- (ii) Find an order-preserving embedding of fields from the residue field $\mathbb{R}((H))_w$ into $\mathbb{R}((H))$.

(Hint: You may find it useful to identify $\mathbb{R}((H))$ with $K((H_1))$ via the isomorphism ψ . Moreover, recall Exercise 11.1.)

- (d) Describe explicitly the residue map

$$\mathbb{R}((H))_w \rightarrow \mathbb{R}((H))_w, s \mapsto sw.$$

Submission:

Please hand in your solutions by **Tuesday, 14 July 2026, 10:00h** (postbox 17).