Nonlinear Waves and their Stability

Konstanz, 31 May – 2 June 2012

Designed to enable interactions between senior and junior scientists, this meeting is devoted to the study of nonlinear waves, notably reagrding their stability. A special emphasis is on internal water waves. The workshop is organized by Heinrich Freistühler (Department of Mathematics), its first day jointly with Frank Peeters (Limnological Institute).

SCHEDULE

Thursday 31 May 2012

9.00 – 9.50	K. Lamb (Waterloo)
10.00 – 10.50	M. Stastna (Waterloo)
11.30 – 12.20	H. van Haren (Den Burg)
14.00 – 14.50	M. Preusse (Konstanz)
15.30 – 16.20	A. Klaiber (Konstanz)
16.30 – 17.20	A. Kamchatnov (Moscow)
19.00	CONFERENCE DINNER at *Schwedenschänke*, isle of MAINAU

Friday 1 June 2012

9.00 – 9.50	S. Benzoni-Gavage (Lyon)
10.00 – 10.50	J. Höwing (Konstanz)
11.30 – 12.20	J. Wächtler (Konstanz)
14.00 – 14.50	P. Szmolyan (Vienna)
15.30 – 16.20	S. Kawashima (Fukuoka)
16.30 – 17.20	P. Secchi (Brescia)

Saturday 2 June 2012

9.00 – 9.50	I. Hashimoto (Kanazawa)
10.00 – 10.50	A. Szepessy (Stockholm)
11.30 – 12.20	M. Kotschote (Konstanz)

Talks on Thursday, May 31, and Friday, June 1, take place in V 1001.
On Saturday, June 2, we meet in G 307.

ABSTRACTS

Thursday, 9:00-9:50: **Kevin Lamb (Waterloo)**

Internal Solitary Waves on the Continental Shelf and their Breaking Mechanisms

Internal solitary-like waves are ubiquitous energetic features of the worlds continental slope/shelf regions. They have a significant impact on the mixing of heat and nutrients, hence playing an important role in biogeochemical processes. In this talk I will present an overview of these waves, discussing observations, theory and numerical simulations. The focus will be on their breaking/dissipation mechanisms including instabilities in the bottom boundary layer beneath ISWs, shear instabilities in the interior of the wave column and breaking due to overturning.

Thursday, 10:00-10:50: **Marek Stastna (Waterloo)**

Large amplitude internal waves and their interaction with the boundary layer

Horizontally propagating internal waves are widely observed in both the coastal and deep ocean as well as medium to large-sized lakes. Due to their nearly solitary nature such waves provide a means to transport energy and momentum over long distances. Moreover, due to their nonlinear nature, they provide a means to transport mass. In this talk I will begin by reviewing the developments in the mathematical description of fully nonlinear internal solitary waves, and internal topographically trapped waves. Both types of waves are solutions of the Dubreil-Jacotin-Long (DJL) equation which is a single, strongly nonlinear elliptic equation that is equivalent to the stratified, incompressible Euler equations. I will discuss what sets the upper bound on the amplitude of the waves and will discuss the generation of large waves by a variety of physical mechanisms, as well as a variety of consequences the properties of large amplitude internal solitary waves have for physics and biology. In the second half of the talk I will consider a structural perturbation of the DJL theory, namely the case of a viscous boundary layer beneath the wave. I will discuss the state of the theory of instabilities in the footprint of the wave, and will give numerical examples of both global and convective instabilities for waves propagating over a flat bottom and topography. Particular attention will be paid to the distinction between instabilities that are restricted to the near boundary region and those that affect the main wave. I will conclude with some speculation about future work.

Thursday, 11:30-12:20: **Hans van Haren (Den Burg)**

Quantifying internal wave – turbulence above deep ocean topography

The stable vertical density stratification in the ocean extends almost from surface to bottom and can be found virtually anywhere, although in different strengths. The stratification supports *internal* waves, which are the main agent for turbulent mixing in the ocean when they break, with relevance for nutrient and sediment redistribution. In the open ocean, internal waves may propagate large distances with only marginal energy dissipation. However, as soon as these waves reach sloping underwater topography, they can develop into highly nonlinear bores that regularly become unstable and break.

Detailed observations using high-sampling rate temperature sensors from various sites above deep ocean topography show ubiquitous internal wave induced turbulence. Quantifying this turbulence in high Reynolds numbers waters reveals energy dissipation rates up to $10^{-4}\ Wkg^{-1}$ in a 50 m large breaking wave that passes the sensors in a few minutes. Such bore-like waves occur once or twice in a tidal period, and all have different energy release depending on the precise development stage and propagation direction when they pass the sensors. Averaged over several tidal periods the amount of turbulence generated over a seamount is 100 times larger than turbulent mixing in the ocean interior. Extrapolating, this implies that internal wave breaking above topography can generate sufficient turbulence to maintain the tidal and advection-diffusion balances of the entire ocean.

Thursday, 14:00-14:50: **Martina Preusse (Konstanz)**
Observations of internal solitary waves in Lake Constance

Long-term temperature measurements in the centre of a sub-basin of Lake Constance, Lake Überlingen, reveal the frequent occurrence of internal solitary wave trains. More than 200 internal solitary wave trains were observed over six years in seasonally varying stratifications. The seasonally averaged wave properties of the leading internal solitary waves were compared with prototypes obtained numerically from the Dubreil-Jacotin-Long equation. The results show a good correspondence between data and models. The passage of about 25 % of these wave trains was accompanied by temperature inversions in the water column indicating wave breaking. The occurrence of such inversions during the passage of the leading solitary wave was well predicable from the parameters amplitude and propagation depth of the leading wave, propagation depth being the rest height of the isotherm undergoing maximum displacement. Amplitude and stability of the leading solitary waves were also compared with the limiting amplitude and breaking mechanisms obtained from simulations with the Dubreil-Jacotin-Long equation. This comparison indicates that the simulated limiting amplitude of the internal solitary waves provide an excellent prediction of the critical wave height above which these waves break in the field. Comparisons of the occurrence of temperature inversions at two deep-water study sites located orthogonal to the propagation direction of the waves furthermore suggest that such breaking events are not local events, but extend over the entire width of the internal solitary wave front. (Joint work with Marek Stastna, Heinrich Freistühler and Frank Peeters)

Thursday, 15:30-16:20: **Andreas Klaiber (Konstanz): A spatial-dynamics approach to the eigenvalue problem of internal travelling waves in stratified fluids**

Fluidic media that are stratified according to varying density, as for example large natural water bodies like oceans and lakes, typically permit the generation and propagation of so-called *internal waves*, which correspond to a time-dependent displacement of fluid elements within central parts of the fluid body, as opposed to surface waves.

A pertinent mathematical model for the description of internal waves is provided by the heterogeneous incompressible Euler equations, posed on the channel $\{(x,y) : x \in \mathbb{R}, 0 < y < 1\}$, where the density stratification of the fluid at rest is known. Contrasting with a rich theory for the existence of internal *travelling* waves and the development of methods for computing these waves, their stability analysis has received less attention at a rigorous mathematical level. In the talk, we present a new approach to the investigation of stability of internal travelling waves in a stratified fluid which consists in (i) a spatial-dynamics formulation of an associated eigenvalue problem, and (ii) a particular way of addressing the eigenvalue problem by applying an Evans function technique to this formulation.

To accomplish (i), we show that the eigenvalue problem of the Euler equations at a travelling wave of speed c as reference state can be written as an infinite-dimensional dynamical system,

$$W'(\xi) = \mathbb{A}(\xi;\kappa,c)W(\xi), \tag{EVP}$$

where $\kappa \in \mathbb{C}$ is the spectral parameter and the horizontal coordinate $\xi = x + ct$ assumes the role of the evolution parameter.

To achieve (ii), we show how this problem (EVP) can be approximated by a sequence of ordinary differential equations,

$$\hat{W}_N'(\xi) = \hat{\mathbb{A}}_N(\xi;\kappa,c)\hat{W}_N(\xi), \quad \text{for } N = 0,1,2,\ldots, \tag{EVP$_N$}$$

which exhibit the notable property of *consistent splitting.* This feature allows to examine the presence of unstable eigenvalues by means of the Evans function – an analytic function defined on the right half-plane whose zeros correspond, under certain assumptions, to eigenvalues – which has been applied successfully to stability questions for various waves, such as shock waves in hyperbolic systems of conservation laws, travelling waves in reaction-diffusion systems and solitons in the generalized Korteweg-deVries equation.

Thursday, 16:30-17:20: **Anatoly Kamchatnov (Moscow)**
Dispersive shock waves: theory and applications

Generation of solitons is a common phenomenon in nonlinear physics which has found many applications. Recently, much attention has been attracted to dispersive shock waves which have been observed in experiments on evolution of Bose-Einstein condensates and other nonlinear media with strong dispersive properties. Dispersive shock waves can be considered as dense lattices of solitons or, in other words, as nonlinear modulated waves which connect two regions of smooth flows with different parameters, and in this sense dispersive shock waves replace viscous shocks well-known in physics of waves in viscous media. In this talk, a short introduction to the theory of dispersive shocks based on the Whitham approach to nonlinear waves modulations will be presented. Generalization of such well-known notions of the linear modulation theory as *group velocity* and *dispersive spreading* of wave packets to the nonlinear case is quite nontrivial and demands development of appropriate mathematics. The general method will be illustrated by several examples of applications of the theory to description of such real phenomena as undular bores in surface and internal water waves, dispersive shocks observed in evolution of clouds of Bose-Einstein condensate and in evolution of light beams in nonlinear optics.

Friday, 9:00-9:50: **Sylvie Benzoni-Gavage (Lyon)**
Traveling waves in the Euler–Korteweg system

The Euler–Korteweg system is a third-order system of conservation laws that takes into account capillary effects in compressible fluids. Depending on the pressure law, it admits more or less rich families of homoclinic/periodic traveling wave solutions. The purpose of the talk is to review the state of the art regarding the stability of those waves. Special attention will be devoted to periodic ones, in connection with Whitham's modulated equations.

Friday, 10:00-10:50: **Johannes Höwing (Konstanz): Stability of solitary waves in generalized Korteweg-de Vries and Euler-Korteweg / Boussinesq equations**

We show that solitary waves for the generalized Korteweg-de Vries equation and for the generalized Boussinesq equation (the p-system endowed with capillarity) are stable if the flux function p satisfies

$$p'' > 0 \qquad \text{and} \qquad p''' \leq 0.$$

While $p'' > 0$ alone suffices for the stability of waves of sufficiently small amplitude, obvious examples show that $p''' \leq 0$ cannot be omitted in the general case. In particular, the generalized Boussinesq equation with

$$p(v) = kv^{-\gamma} \qquad \text{with } \gamma \geq 1, \ \ k > 0$$

describes the flow of an inviscid isothermal ideal (barotropic) fluid with capillarity. In this talk, we present the following new stability results:
Theorem 1. Consider the generalized Korteweg-de Vries (gKdV) equation with a smooth function p satisfying $p'' > 0$ and $p''' \leq 0$. Then any solitary wave is stable.
Theorem 2. Consider the generalized Boussinesq equation with $p : \mathbb{R} \to \mathbb{R}$ or $p : (0, \infty) \to \mathbb{R}$ satisfying $p' < 0$, $p'' > 0$ and $p''' \leq 0$. Then any solitary wave is stable.
These results complement the findings of Bona, Souganidis and Strauss 1987, and Bona and Sachs 1988, respectively; the only overlap of Theorems 1 and 2 with those consisting exactly of the quadratic nonlinearity $p''' \equiv 0$. Note, however, that Theorems 1 and 2 are not restricted to pure power laws. Furthermore, we will extend some of our results to the case of non-constant capillarity in the Euler-Korteweg equation.

Friday, 11:30-12:20: **Johannes Wächtler (Konstanz)**
Spectral stability of small shock waves associated with a degenerate mode

We study the spectral stability of small shock waves in a system of viscous conservation laws in the case that the underlying mode is not genuinely nonlinear. Suitably scaled, the associated eigenvalue problem has a slow-fast structure. Using geometric singular perturbation theory, we will analyze the dynamics in the zero-amplitude limit, and prove that the stability problem essentially reduces to the simpler problem of a shock wave in the scalar equation

$$u_t + (u^3)_x = u_{xx}.$$

In order to receive a full picture of the dynamics, we use a blow-up in parameter space.

Friday, 14:00-14:50: **Peter Szmolyan (Vienna): Geometric analysis of Lagerstrom's model problem for low Reynolds number flow past a sphere**

Lagerstrom's model problem is a classical singular perturbation problem which was introduced to illustrate the ideas and subtleties involved in the analysis of viscous flow past a solid at low Reynolds number by the method of matched asymptotic expansions. We present a geometric analysis of the corresponding boundary value problem based on methods from the theory of dynamical systems, in particular invariant manifold theory. As an essential part of the dynamics takes place near a line of non-hyperbolic equilibria, we introduce a blow-up transformation to resolve these singularities. This approach leads to a constructive proof of existence and local uniqueness of solutions and to a better understanding of the singular perturbation nature of the problem. In particular, the phenomenon of logarithmic switchback, i.e. the unexpected occurence of logarithmic terms in the asymptotic expansion, is explained as a nonlinear resonance effect.

Friday, 15:30-16:20: **Shuichi Kawashima (Fukuoka)**
Dissipative structure for symmetric hyperbolic systems

We discuss the decay property for a class of symmetric hyperbolic systems with relaxation. The Shizuta-Kawashima stability condition gives the characterization of the standard dissipative structure for systems with symmetric relaxation matrices. Recently, we found several intereting systems with non-symmetric relaxation which have different dissipative structure. In this talk, we discuss these examples and report the recent progress on the stability theory for a class of symmetric hyperbolic systems.

Friday, 16:30-17:20: **Paolo Secchi (Brescia)**
Stability of the free plasma-vacuum interface

We consider the free boundary problem for the plasma-vacuum interface in ideal compressible magnetohydrodynamics (MHD). In the plasma region the flow is governed by the usual compressible MHD equations, while in the vacuum region we consider the pre-Maxwell dynamics for the magnetic field. At the free-interface we assume that the total pressure is continuous and that the magnetic field is tangent to the boundary. The plasma density does not go to zero continuously at the interface, but has a jump, meaning that it is bounded away from zero in the plasma region and it is identically zero in the vacuum region. Under a suitable stability condition satisfied at each point of the initial discontinuity, we prove the well-posedness of the linearized problem in conormal Sobolev spaces. (Joint work with Y. Trakhinin (Novosibirsk).

Saturday, 9:00-9:50: **Itsuko Hashimoto (Kanazawa)**
Asymptotic behavior of radially symmetric solutions for Burgers equation on exterior domains

We study the large-time behavior of the radially-symmetric solution for Burgers equation on the outer space of multi dimensional space, where the data of boundary and at the far field are prescribed. This problem is reduced to the problem on the half line and we consider the case where the corresponding Riemann problem for the hyperbolic part admits the rarefaction wave.
It is shown that the asymptotic states are divided dependent on the signs of the characteristic speeds on the boundary and the far field. Furthermore, we also derive the time convergence rate. The proof is given by a standard L^2-energy method and time weighted energy method.

Saturday, 10:00-10:50: **Anders Szepessy (Stockholm)**

How accurate is molecular dynamics?

Molecular dynamics is used to determine constitutive relations in continuum mechanics, such as the pressure as a function of density, velocity and internal energy in a compressible fluid. Shnirelman showed 1974 how classical ergodic dynamics approximates Schrödinger observables, which was the start for the activity on quantum ergodicity. Using characteristics, I will show you how Shnirelman's ideas can be used to study the accuracy of molecular dynamics systems.

Saturday, 11:30-12:20: **Matthias Kotschote (Konstanz)**

Strong solutions to the Navier-Stokes equations for a compressible fluid of Allen-Cahn type

In this talk, we study the "Navier-Stokes-Allen-Cahn" system, a combination of the compressible Navier-Stokes equations with an Allen-Cahn phase field description. This model admits of describing two-phase patterns in a flowing liquid including phase transformations. The purpose is to show existence and uniqueness of local strong solutions for arbitrary initial data.
