EWM-Graz 2018: Real Algebraic Geometry in Action

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Abstract:

In this talk we deal with the following general version of the classical moment problem: when can a linear functional on a unital commutative real algebra A be represented as an integral w.r.t. a Radon measure on the character space of A equipped with the Borel sigma-algebra generated by the weak topology? In a joint work with Salma Kuhlmann, Tobias Kuna and Patrick Michalski, we approach this problem by constructing the character space X(A) as a projective limit of a certain family of Borel measurable spaces and so by considering on X(A) the associated cylinder sigma-algebra beside the Borel one. This allows us to obtain representations of linear functionals, which are positive on sum of squares in A and fulfill certain quasi-analytic bounds, as integrals w.r.t. measures defined on the cylinder sigma-algebra on X(A). Combining this result with the well-known Prokhorov theorem, we get extensions of such measures to the Borel sigma-algebra and, hence, generalize to infinitely (even uncountably) generated algebras some of the classical theorems for the moment problem such as the ones by Nussbaum and Putinar. Our results apply in particular to the case when A is the polynomial algebra in an arbitrary number of variables, providing alternative proofs of some recent results for this instance of the moment problem and offering at the same time a unified setting which enables comparisons.