

EWM-Graz 2018: Real Algebraic Geometry in Action

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" Factorizations and Positivstellensätze in Free Noncommutative Analysis "

Abstract:

Factorization theorems, such as Fejer--Riesz factorization of positive trigonometric polynomials and Kolmogorov--Aronszajn factorization of positive kernels, have longed played a central role in one-variable complex analysis and operator theory. Multivariable analogues came to prominence starting with the work of Agler in the late 1980s and early 1990s. There are close similarities between these results and the Positivstellensätze of real algebraic geometry, including the analytic proofs that use a separation argument followed by a GNS-type construction. There are also some important differences: the setting is that of analytic functions rather than that of polynomials, and one is interested in hermitian squares of analytic functions leading to a possibility of evaluation on tuples of commuting matrices of all sizes (or even tuples of commuting operators on a Hilbert space) rather than just scalar evaluations. I will present a glimpse of these parallel universes, and discuss some recent joint work with J. Ball and G. Marx in the setting of free noncommutative function theory.