LICENCE BIOLOGIE MATHÉMATIQUES 2008-2009

Corrigé de la feuille 2.

1. $$\frac{\partial f(x,y,z)}{\partial x} = \frac{2x}{1+y^2}$$

$$\frac{\partial f(x,y,z)}{\partial y} = (x^2+z^2)(-1)\frac{1}{(1+y^2)^2}\,2y = \frac{-2y(x^2+z^2)}{(1+y^2)^2}$$

$$\frac{\partial f(x,y,z)}{\partial z} = \frac{2z}{1+y^2}$$

Donc $\frac{\partial f}{\partial x}(1,1,1) = 1$,

$\frac{\partial f}{\partial y}(1,1,1) = -1$ et

$\frac{\partial f}{\partial z}(1,1,1) = 1$.

2. $$\frac{\partial f}{\partial x}(x,y) = \frac{1}{x^2+y^2}\cdot 2x = \frac{2x}{x^2+y^2}$$

$$\frac{\partial f}{\partial y}(x,y) = \frac{1}{x^2+y^2}\cdot 2y = \frac{2y}{x^2+y^2}$$

$$\frac{\partial^2 f}{\partial x^2}(x,y) = \frac{\partial}{\partial x}\left(\frac{2x}{x^2+y^2}\right) = \frac{(x^2+y^2)\cdot 2 - 2x\,2x}{(x^2+y^2)^2}$$

$$= \frac{-2x^2+2y^2}{(x^2+y^2)^2} = 2\,\frac{y^2-x^2}{(x^2+y^2)^2}$$

Pour des raisons de symétrie on obtient

$$\frac{\partial^2 f}{\partial y^2}(x,y) = 2\,\frac{x^2-y^2}{(x^2+y^2)^2}.$$

Enfin on calcule

$$\frac{\partial^2 f}{\partial x \partial y}(x,y) = \frac{\partial}{\partial x}\left(\frac{2y}{x^2+y^2}\right) = 2y\,\frac{-1}{(x^2+y^2)^2}\,2x$$

$$= \frac{-4xy}{(x^2+y^2)^2}$$

Pour le laplacien on obtient

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0.$$

3. $$\frac{\partial}{\partial x}T(x,y) = \frac{\partial}{\partial x}\left(\frac{60}{1+x^2+y^2}\right) = 60\,\frac{-1}{(1+x^2+y^2)^2}\,2x$$

$$= \frac{-120x}{(1+x^2+y^2)^2}$$

$$\frac{\partial}{\partial y}T(x,y) = \frac{-120y}{(1+x^2+y^2)^2}$$

$$\frac{\partial T}{\partial x}(2,1) = \frac{-240}{(1+5)^2} = \frac{-240}{36} = \frac{-20}{3}$$

$$\frac{\partial T}{\partial y}(2,1) = -\frac{10}{3}$$

Taux de variation de température par rapport à la distance au point $(2,1)$ dans la direction de l'axe des x: $-\frac{20}{3}$

des y: $-\frac{10}{3}$

4. (a) $\ln P = (\ln k) + (\ln T) - \ln(V)$

(b) $\frac{\Delta P}{P} = \Delta \ln P \sim \frac{\partial \ln(P)}{\partial T} \Delta T + \frac{\partial \ln(P)}{\partial V} \Delta V$

Or $\frac{\partial \ln(P)}{\partial T} = \frac{1}{T}$ et $\frac{\partial \ln(P)}{\partial V} = -\frac{1}{V}$ et

donc $\frac{\Delta P}{P} \sim \frac{\Delta T}{T} - \frac{\Delta V}{V}$

d'où $\left|\frac{\Delta P}{P}\right| \lessapprox \left|\frac{\Delta T}{T}\right| + \left|\frac{\Delta V}{V}\right|$

(c) $\left|\frac{\Delta P}{P}\right| \lessapprox 0{,}007$

5. (a) $\ln S = (\ln(71{,}84)) + 0{,}725 \ln T + 0{,}425 \ln P$

(b) $\frac{\Delta S}{S} = \Delta \ln S \sim \frac{\partial \ln(S)}{\partial T} \Delta T + \frac{\partial \ln(S)}{\partial P} \Delta P$

$\frac{\partial \ln(S)}{\partial T} = \frac{0{,}725}{T}$; $\frac{\partial \ln(S)}{\partial P} = \frac{0{,}425}{P}$

donc $\frac{\Delta S}{S} \sim 0{,}725 \frac{\Delta T}{T} + 0{,}425 \frac{\Delta P}{P}$

(c) $\left|\frac{\Delta S}{S}\right| \lessapprox 0{,}725 \cdot 0{,}001 + 0{,}425 \cdot 0{,}005$

$= 0{,}00285$