We consider a finite system of non-strict real polynomial inequalities (SPI). Its Lasserre relaxation of degree d is a certain natural linear matrix inequality (LMI) in the original variables and one additional variable for each nonlinear monomial of degree at most d. This LMI defines a spectrahedron that projects down to a convex semialgebraic set containing the solution set of the SPI. In the best case, the projection equals the convex hull of the solution set of the SPI. We say that the Lasserre hierarchy *eventually becomes exact* if this is the case for all sufficiently large d.

We suppose that the SPI satisfies the *Archimedean* condition which is nearly equivalent to its solution set being compact in the following sense: An Archimedean SPI has compact solution set. Conversely, an SPI with compact solution set can be made Archimedean by adding certain appropriate redundant inequalities.

In [1], Kriel and myself showed that if the solution set of the SLI "bulges outwards" with positive curvature, then the Lasserre hierarchy very often eventually becomes exact. The proof combines ingredients from several areas:

- Real closed fields and real quantifier elimination from real algebraic geometry,
- pure states and separation theorems from functional analysis (applied to vector spaces over real closed fields which are considered as real vector spaces),
- Lagrange multipliers and the Karush-Kuhn-Tucker theorem from convexity (after being transferred to real closed fields),
- the finiteness theorem from first order logic (which follows for example from Gödel's completeness theorem).

The major drawback of this theorem is that it does in general not allow for linear constraints in the SLI . In the talk, we give an example of an SLI with one quartic inequality in two variables that defines the disjoint union of two disks in the plane of different radii and one linear inequality defining an affine half space that cuts out part of the bigger disk but precisely preserves the smaller disk. In this example, the Lasserre hierarchy does *not* eventually become exact.

With completely different and more traditional techniques, Kriel and myself showed in [2] a similar second theorem whose advantage is that it allows for linear constraints and more generally constraints satisfying a certain "relative sums-of-squares concavity condition". However, this alternative theorem supposes the solution set of the SLI to be convex.

Neither of our two theorems on SLIs seems to be accessible to the techniques we used to prove the respective other theorem.

Now let additionally a polynomial objective function be given, i.e., consider a polynomial optimization problem (POP). Its Lasserre relaxation of degree d is now a semidefinite program (SDP). In the best case, the optimal values of the POP and the SDP agree. In [1], Kriel and myself proved that this often happens if the relaxation degree exceeds some bound that depends on the constraints of the POP and certain characteristicae of the objective like the mutual distance of its global minimizers on the feasible region.

References

[1] T.-L. Kriel, M. Schweighofer, *On the Exactness of Lasserre Relaxations and Pure States Over Real Closed Fields*, Foundations of Computational Mathematics, Online First (2019), 100–120.

[2] T.-L. Kriel, M. Schweighofer, *On the exactness of Lasserre relaxations for compact convex basic closed semialgebraic sets*, SIAM J. Optim. **28** (2018), 1796–1816.