ERRATUM TO: INFEASIBILITY CERTIFICATES FOR LINEAR MATRIX INEQUALITIES

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ABSTRACT. We fix a minor technical problem in our Oberwolfach preprint [KS].

There is a flaw in the proof of [KS, Lemma 3.5.1]. The problem is at the end of the induction step where we say that $g \in (\ell_2, \ldots, \ell_t)$. One can only conclude that $g \in (\ell_2(\ell, X_2, \ldots, X_n), \ldots, \ell_t(\ell, X_2, \ldots, X_n))$. Indeed there is a trivial counterexample to [KS, Lemma 3.5.1]: $d = 1$, $f = X_2$, $\ell_1 = X_1$, $\ell_2 = X_1 + 1$.

However, this does not affect any of the (other) results of [KS]. To show this, we formulate a correct version of the lemma in question:

Lemma 3.5.1. *Suppose $d \in \mathbb{N}$, $f \in \mathbb{R}[\underline{X}]_d$ and $\ell_1, \ldots, \ell_t \in \mathbb{R}[\underline{X}]_1$ are linear polynomials such that $f \in (\ell_1, \ldots, \ell_t)$. Then at least one of the following is true:*

(a) *there exist $p_1, \ldots, p_t \in \mathbb{R}[\underline{X}]_{d-1}$ such that $f = p_1\ell_1 + \cdots + p_t\ell_t$;*

(b) *there are $\lambda_1, \ldots, \lambda_t \in \mathbb{R}$ such that $\lambda_1\ell_1 + \cdots + \lambda_t\ell_t = 1$.*

Proof. Suppose that (b) is not fulfilled. Then we may assume by Gaussian elimination and after renumbering the variables that $\ell_i = X_i - \ell_i'$ where $\ell_i' \in \mathbb{R}[X_{i+1}, \ldots, X_n]_1$. With this additional hypothesis, we prove (a) by induction on $t \in \mathbb{N}_0$ exactly like in [KS] (with $\ell := \ell_1'$). □

Now it is enough to correct the proof of [KS, Theorem 3.5.2] as follows:

Corrected proof of Theorem 3.5.2: [...] Second, in Lemma 3.5.1 applied to $\ell_1, \ldots, \ell_{i-1}$ $(i \in \{1, \ldots, n+1\})$ ~~the ℓ_i are assumed to be non-constant but can be allowed to equal zero.~~ case (b) might happen. But ~~if some $\ell_i \neq 0$ is constant,~~ then we may set $\ell_{i+1} = \cdots = \ell_n = 0$ and $S_{i+1}' = \cdots = S_n' = S = 0$. [...] □

REFERENCES

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Date: March 4, 2012.